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THESIS

DESIGN OF A MULTI-DOF TUNED MOUNTING FIXTURE
FOR THE NAVY'S MEDIUMWEIGHT SHOCK MACHINE

by

David M. Cox

June 1993

Thesis Advisor:

Y.S. Shin

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A near miss underwater explosion can significantly damage improperly shock hardened combat systems equipment and render the ship unable to "fight hurt". MIL-S-901D currently requires shock qualifying mediumweight equipment to a "generic" shock excitation on the Navy's Mediumweight Shock Machine (MWSM). This shock excitation is severe, but not always characteristic of the actual ship structure response to an underwater explosion. This study proposes a design modification which will allow using a multi-DOF equipment mounting fixture on the MWSM which can be "tuned" to simulate shipboard shock characteristics based on modal testing and previous ship shock trial data. Equipment qualified in this manner could be highly relied on to survive in battle.					
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DESIGN OF A MULTI-DOF TUNED MOUNTING FIXTURE FOR THE NAVY'S
MEDIUMWEIGHT SHOCK MACHINE

by

David M. Cox
Lieutenant, United States Navy
B.S.M.E., Old Dominion University, 1988

Submitted in partial fulfillment
of the requirements for the degree of

MASTER OF SCIENCE IN MECHANICAL ENGINEERING

from the

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June 1993

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ABSTRACT

A near miss underwater explosion can significantly damage improperly shock hardened combat systems equipment and render the ship unable to "fight hurt". MIL-S-901D currently requires shock qualifying mediumweight equipment to a "generic" shock excitation on the Navy's Mediumweight Shock Machine (MWSM). This shock excitation is severe, but not always characteristic of the actual ship structure response to an underwater explosion. This study proposes a design modification which will allow using a multi-DOF equipment mounting fixture on the MWSM which can be "tuned" to simulate shipboard shock characteristics determined from modal testing or previous ship shock trial data. Equipment qualified in this manner could be highly relied on to survive in battle.

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I. INTRODUCTION

In today's world of modern warfare the U.S. Navy must rely on state of the art combat systems equipment to maintain its fighting advantage over possible adversaries. However, the high-tech nature of this equipment can lead to increased vulnerability to mechanical shock induced failure if not adequately packaged to withstand the severe excitations expected from conventional and nuclear underwater weapons. When exploded in proximity of the ship, these weapons produce an intense pressure wave which is applied over the entire underwater portion of the ship's hull. Although the hull is likely to remain intact, the violent complex shock waves that propagate throughout the ship can sufficiently damage essential equipment to render the ship unable to "fight hurt" and significantly impair its mission integrity.

The Navy currently uses Military Specifications (MIL-S-901D), "Shock Tests, High Impact; Shipboard Machinery, Equipment and Systems, Requirements For" to detail specific shock qualification requirements for shipboard machinery, equipment, systems and structures which are required to resist the effects of mechanical shock. These requirements establish the general shock test criteria and provide the contracting activity a basis for selecting the appropriate testing device based on the weight category of the equipment. The weight

categories defined are; lightweight for an attached weight up to 550 lb; mediumweight for an attached weight up to 7400 lb; and heavyweight for a total weight up to 60,000 lb. The lightweight and mediumweight machines are similar in that the high impact shock is delivered to the attached equipment by use of a hammer and anvil assembly. Heavyweight category equipment is installed onboard a floating platform barge and subjected to shock from an underwater explosive of known charge and standoff geometry. Of the three weight categories listed, this research focuses on the use of the Medium Weight Shock Machine (MWSM) shown in Figure 1.

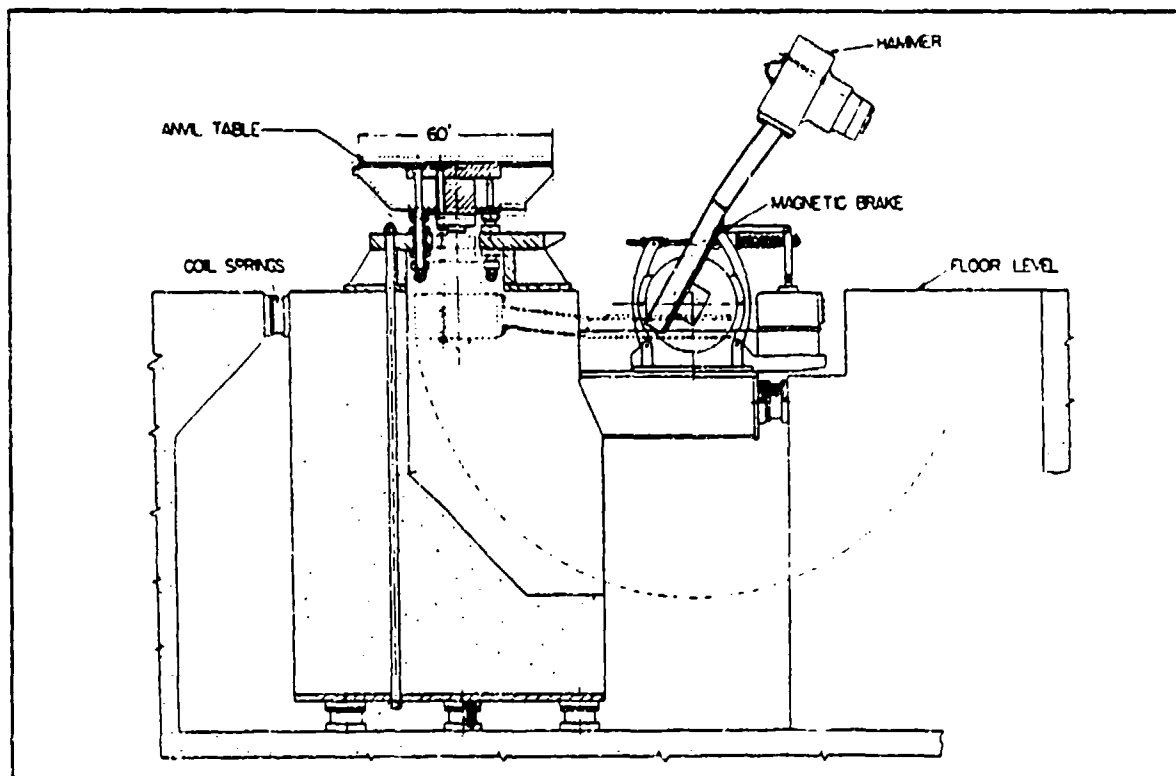


Figure 1. The Navy High-Impact Shock Machine for Mediumweight Equipment (MWSM). Courtesy of Clements(1972).

The MWSM delivers a vertical high impact mechanical shock to the anvil table by means of a 3000 lb swinging hammer. The impact between the hammer and anvil is highly elastic and the energy is controlled by adjusting the height of the hammer above the anvil table prior to release. Required hammer heights are specified in MIL-S-901 based on the total weight attached to the anvil table.

Deck mounted surface ship equipment is normally attached to the anvil table using the standard mounting fixture which is designed to provide a great deal of flexibility in equipment mounting geometry. The standard fixture is shown in Figure 2. Submarine deck mounted equipment is attached using the coil spring soft deck simulator shown in Figure 3. When using the standard fixture, MIL-S-901D specifies the number and type of support channels to be used based on the equipment weight and mounting bolt spacing. As noted by Clements(1972), the arrangement specified in MIL-S-901D was design to keep the calculated maximum stress in the channels below 35,000 psi in a static acceleration field of 50 g's. Although not by design, using this arrangement produces an equipment excitation in the range of 55 Hz to 72 Hz. The soft deck simulator has a natural frequency in the range of 20 Hz to 25 Hz and is designed to simulate the natural deck frequencies of a submarine.

Equipment tested using the standard fixture will be excited predominately at a single frequency in the 55 Hz to 72 Hz range which is not realistic of the excitation aboard ship.

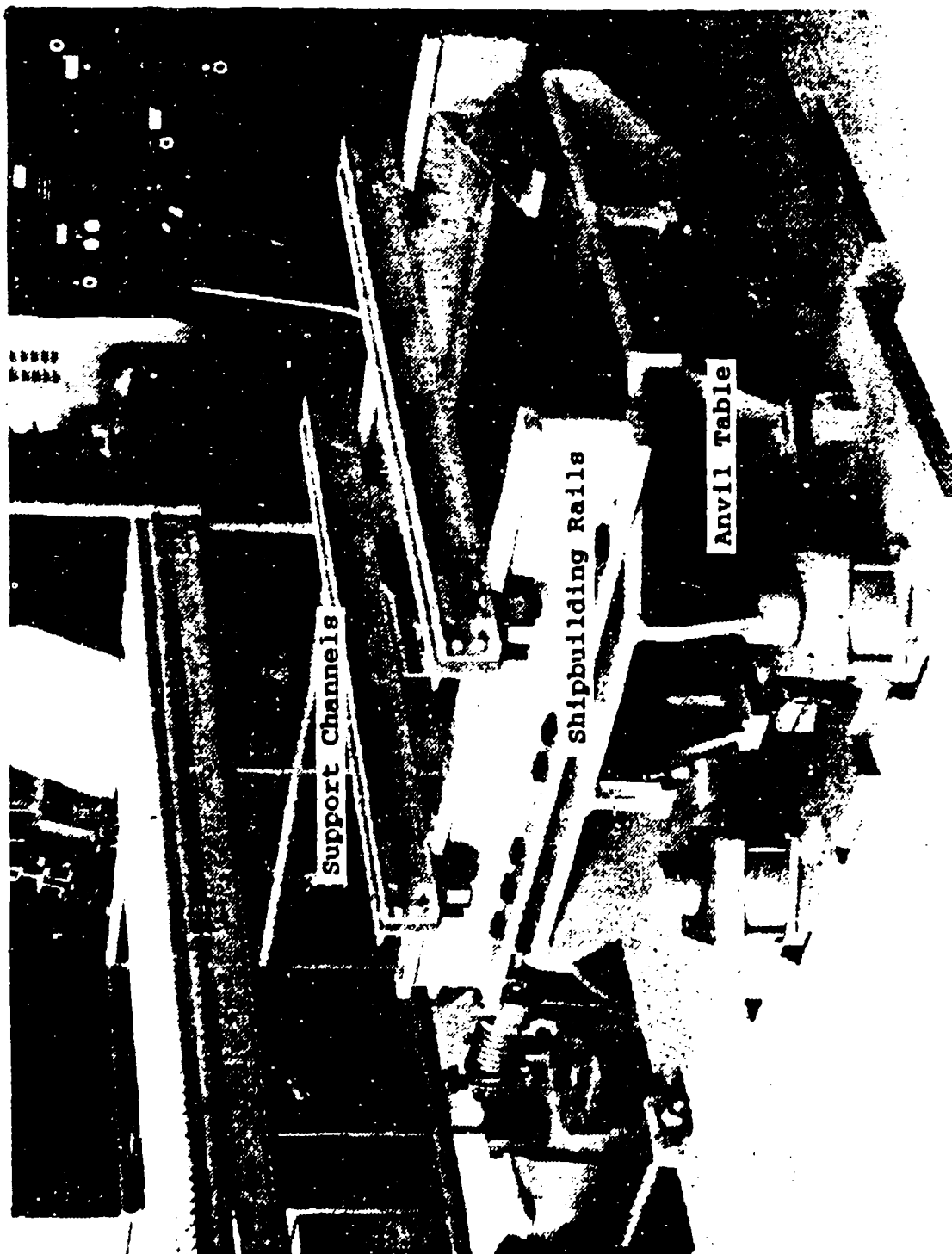


Figure 2. Standard Mounting Fixture for the MWSM. Courtesy of Clements(1972).

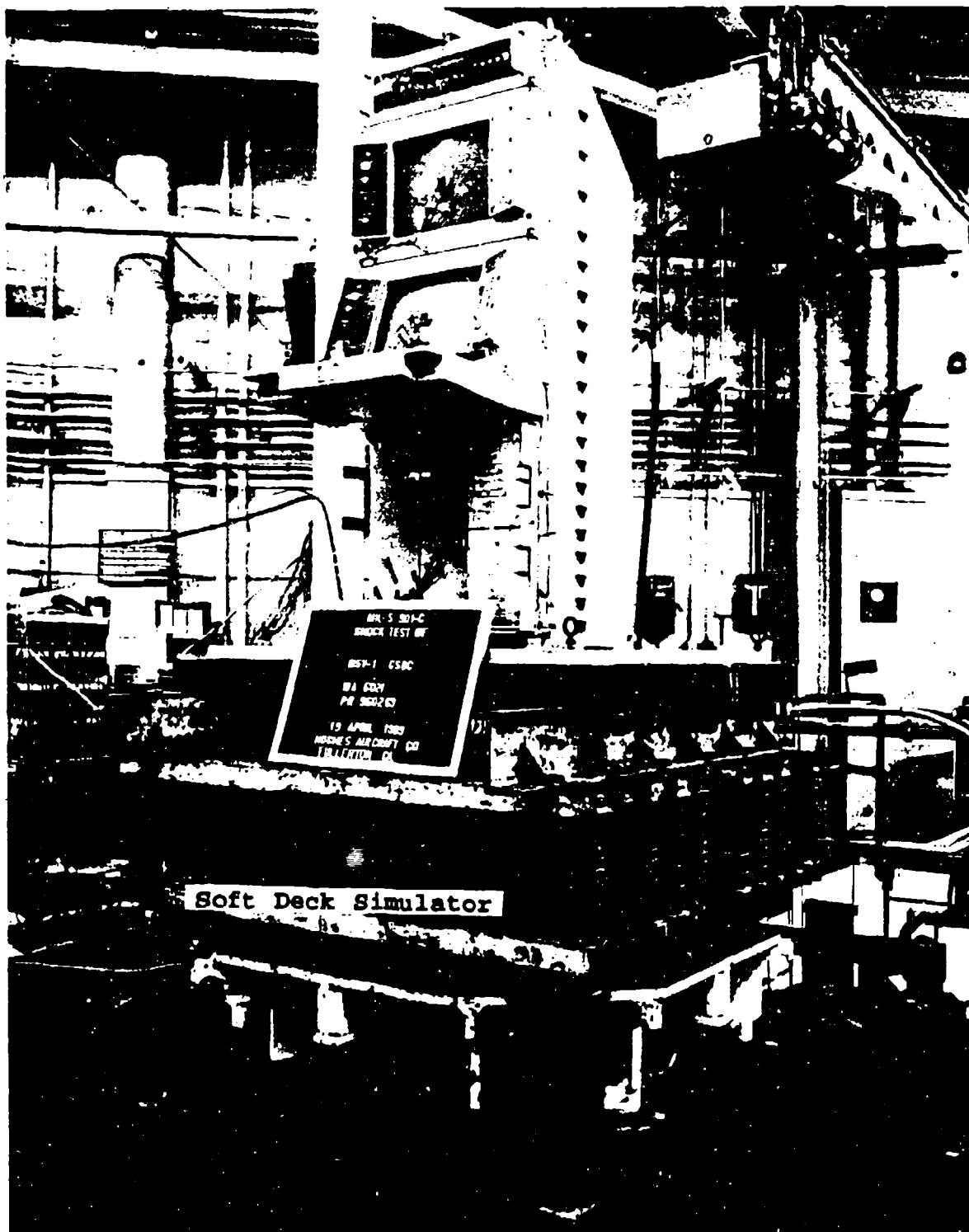


Figure 3. Soft Deck Simulator for MWSM. Courtesy of Hughes Aircraft.

Onboard ship, the excitation will be a complex combination of the natural frequencies of the ship's structure in the location of the equipment installation. Shipboard vibration may excite natural frequencies within the equipment and lead to severe damage that would have not been detected on the MWSM. This introduces the need to develop a replacement for the standard fixture which will produce a shock excitation more realistic of the local shipboard response to underwater explosions.

This study examines the design of a multi-degree-of-freedom fixture to be installed on the MWSM which can be tuned to more accurately simulate shipboard shock excitation of combat systems equipment. The shock spectrum of the excitation input to three representative pieces of equipment modeled in the DDG-51 Class Ship Pre-Shock Trial Analysis performed by Costanzo and Murray (1991) at the Underwater Research Division of David Taylor Research Center (DTRC/UERD) is used to identify the desired excitation response of the fixture. These shock spectra identify the frequency components of the equipment excitation and provide the basis for tuning the fixture. This fixture is proposed as an economical means of providing realistic excitations to combat systems equipments when shock qualifying on the U.S. Navy's MWSM.

II. BACKGROUND PRESENTATION

A. THE NEED FOR A TUNED MOUNTING FIXTURE FOR THE MWSM

In an effort to the increase the survivability of ships in battle, more attention is being focused on the shock qualifications of combat systems equipment installed on ship. To ensure the reliability of this equipment in a mechanical shock environment it must first be "shock qualified" on the appropriate shock machine. For the test to be sufficiently valid, the mechanical shock excitation to the equipment should be simulated as closely as possible to the expected local ship structure response to an underwater explosion.

MIL-S-901D requires that the test item be mounted to the shock machine anvil table in a manner characteristic of its shipboard orientation. For most equipment, this requires using a specified number of support channels and rails which make up the standard fixture (Figure 2). Chalmers and Shaw (1989) note that most users of MIL-S-901D believe that using the standard fixture on the MWSM produces a more severe, all encompassing shock excitation than would actually be experienced onboard ship. They provide evidence in their report that using the standard fixture can be an undertest as well as an overtest since a high frequency mounting will indeed pass higher acceleration levels at higher frequencies, but will not

provide the resonant amplifications generated by a lower frequency fixture. In essence, if the ship's structure excites the equipment at a lower frequency than was supplied by the MWSM, the installed equipment may experience severe resonant vibrations that were not experienced during testing. If a low frequency fixture such as the soft deck simulator were used, it too may be an undertest since it would not excite the higher resonant frequencies that may be present in the equipment.

As noted in Corbell(1992), a finite element transient shock analysis of the DDG-51 Class Deck House was conducted by the Ship Structure and Protection Department of David Taylor Research Center (DTRC/UERD). The preliminary report by Costanzo and Murray (1991) was obtained along with the predicted shock analysis for various weight combat systems equipment located on the 0-3 level of the DDG-51 class ship. From this report, the predicted shock excitations to three representative combat systems equipments were used as the desired response for a tuned fixture on the MWSM. The following equipments selected fall within the weight range of test items normally qualified on the MWSM:

- Radar Receiver Transmitter (RT-1293/SPS-67) 325 lbs
- Beam Programmer (MX-10873/SPY-1D) 1000 lbs
- Radio Frequency Amplifier (AM-7159/SPY-1B) 4600 lbs

The predicted acceleration wave forms and associated shock spectra, analyzed for maximum shock trial severity, for the three equipments are shown in Figures 4 through 6.

The shock spectra which are insensitive to small waveform variations, describe the characteristic frequencies of the shock induced excitations. The shock spectrum is generated by plotting the maximum absolute response of a single-DOF undamped oscillator as a function of its natural frequency when subjected to the base excitation of interest. It is important to note, that when attempting to compare shock motion, the wave "form" in the time domain is much less important than the wave "characteristics" displayed in the shock spectrum (frequency domain). Therefore, when trying to reproduce the shipboard excitation on the MWSM it is not necessary to reproduce the wave form, only the frequency components at proper acceleration amplitude levels. The predicted shock spectra demonstrate that the deck house shock environment will be significantly different than the high energy, single frequency shock excitation produced by using the standard fixture on the MWSM. MIL-S-901D does not require that any particular waveform or spectrum be reproduced, however the need for a tuned mounting fixture for the MWSM clearly exists.

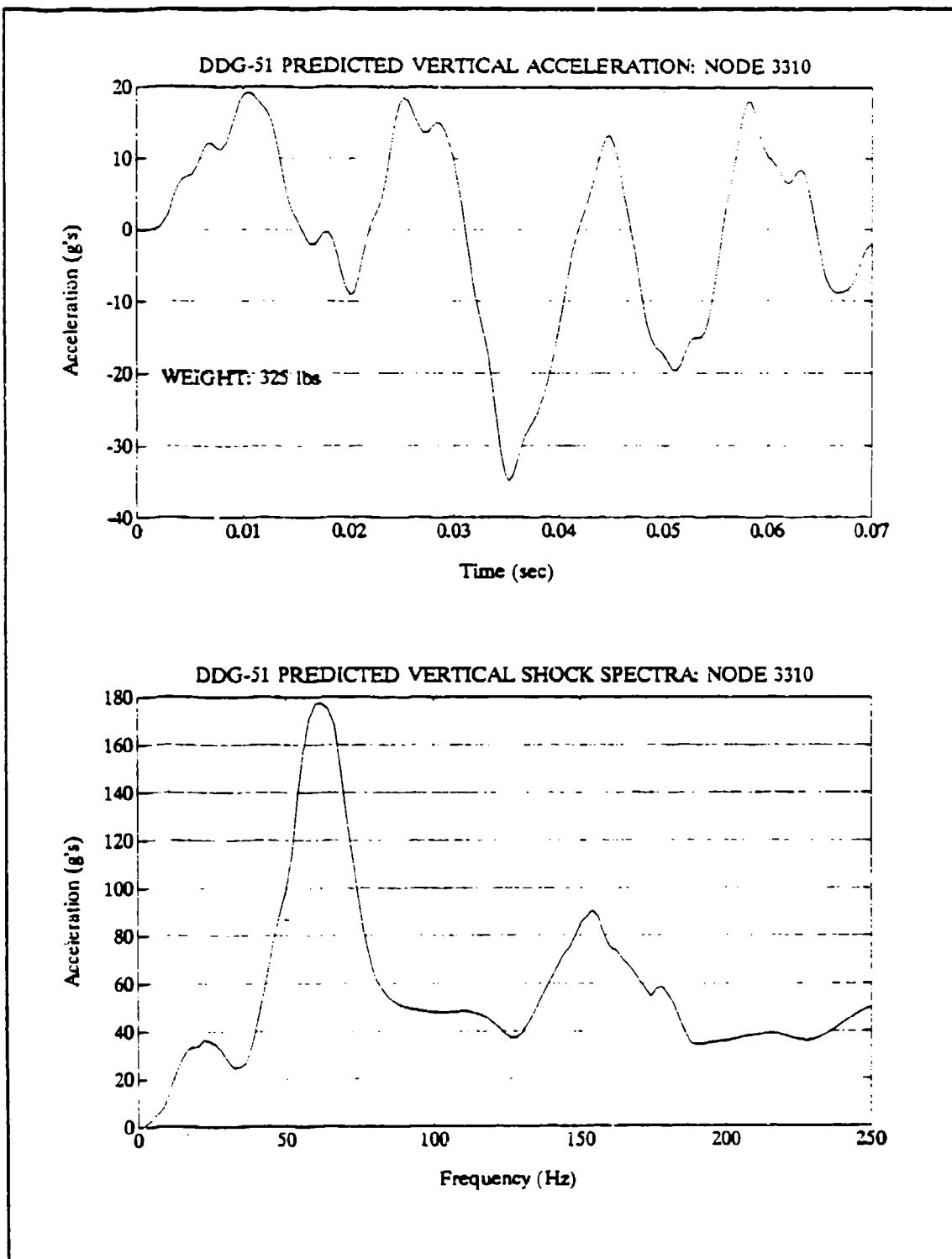


Figure 4. DTRC/UERD Predicted Radar Receiver/Transmitter Vertical Acceleration Wave Form and Shock Spectrum.

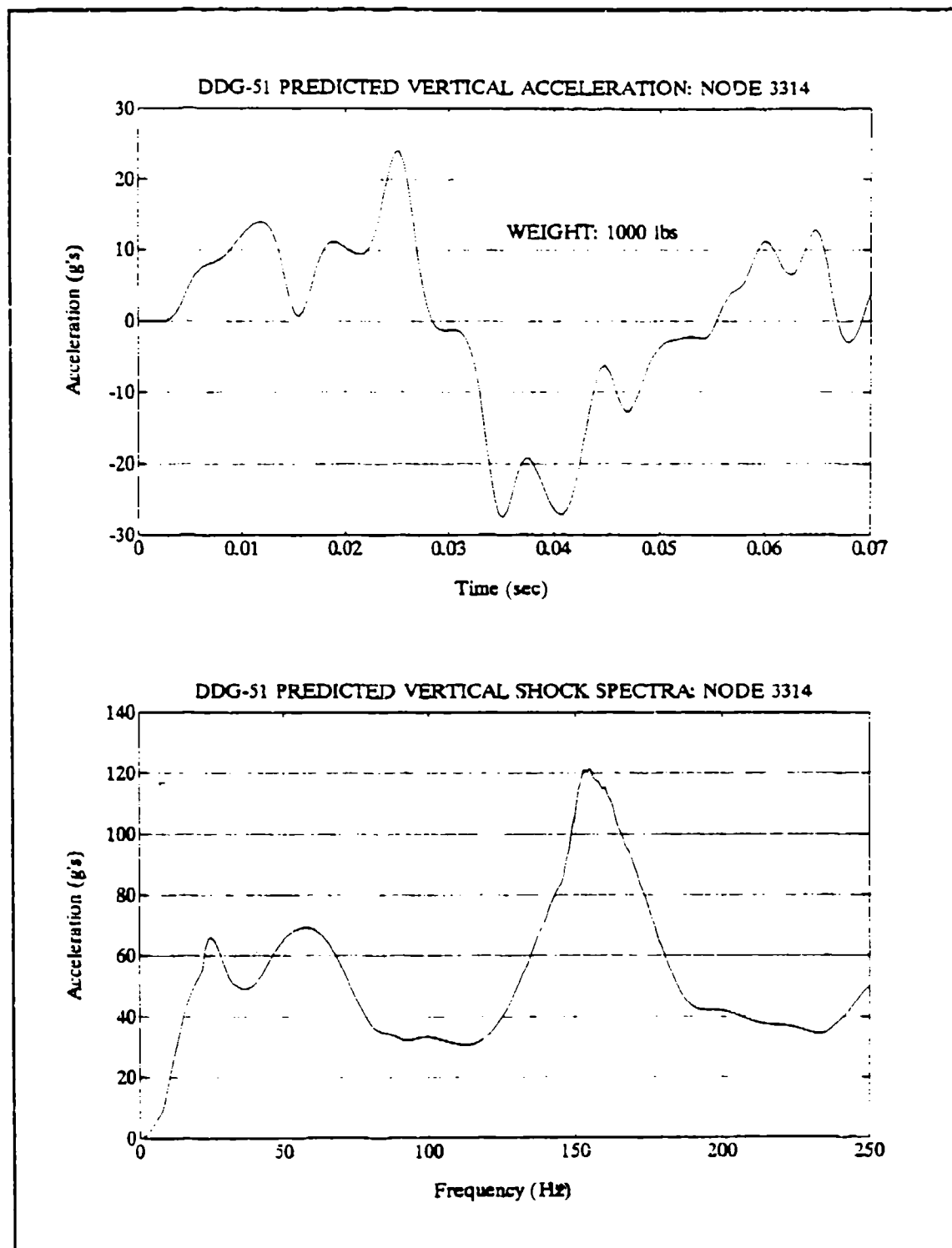


Figure 5. DTRC/UERD Predicted Beam Programmer Vertical Acceleration Wave Form and Shock Spectrum.

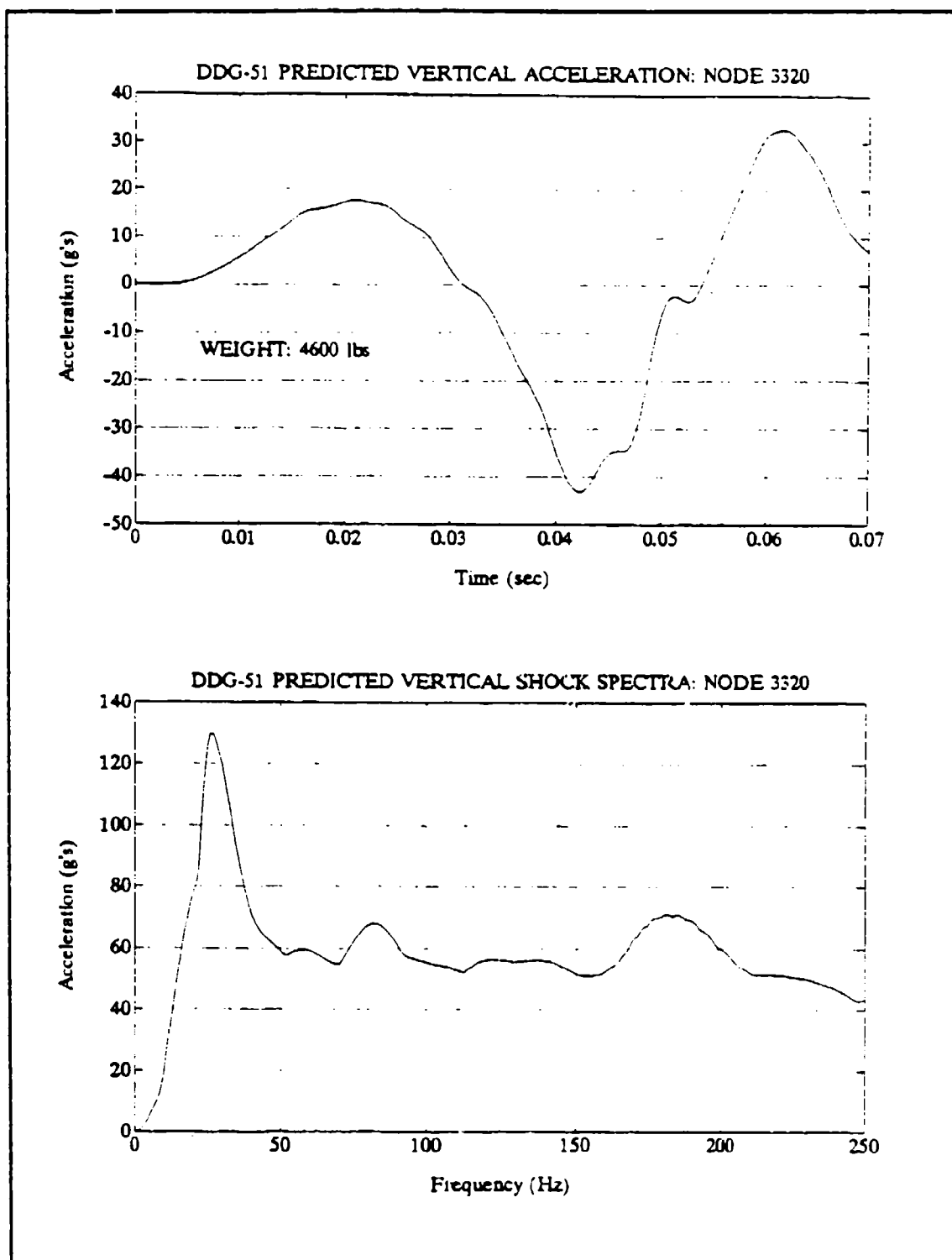


Figure 6. DTIC/JERD Predicted Radio Frequency Amplifier Vertical Acceleration Wave Form and Shock Spectrum.

B. COMPARING SHIPBOARD AND TUNED MOUNTING FIXTURE RESPONSE

Extending the concept developed by Chalmers and Shaw (1989), Corbell proposed that a two degree-of-freedom (DOF) fixture could be designed for the MWSM that would excite an attached piece of equipment with most of the energy concentrated at two resonant frequencies. If the fixture was "tunable", a MIL-S-901D user could refer to UNDEX shock spectra data or modal testing data for a specific area of a ship and tune the fixture to simulate two of the dominant excitation frequencies.

Figure 7 shows a comparison of the DTRC/UERD DDG-51 predicted shock spectrum for a radar receiver/transmitter to the computer modeled shock spectra of the Standard Fixture and two DOF fixture on the MWSM. The DTRC/UERD predictions show significant levels of acceleration amplitudes at 55 Hz and 155 Hz. The modeled Standard Fixture provides a high level of acceleration at 72 Hz but does not supply a sufficient amplitude of acceleration between 130 Hz and 170 Hz to adequately shock qualify the radar with respect to the DTRC/UERD predictions.

The two DOF model was "tuned" to better simulate the shock characteristics predicted for the radar receiver/transmitter. As can be seen from Figure 7, the two DOF model reproduced the equipment accelerations at the characteristic frequencies and amplitudes of the DTRC/UERD predictions. Equipment qualified

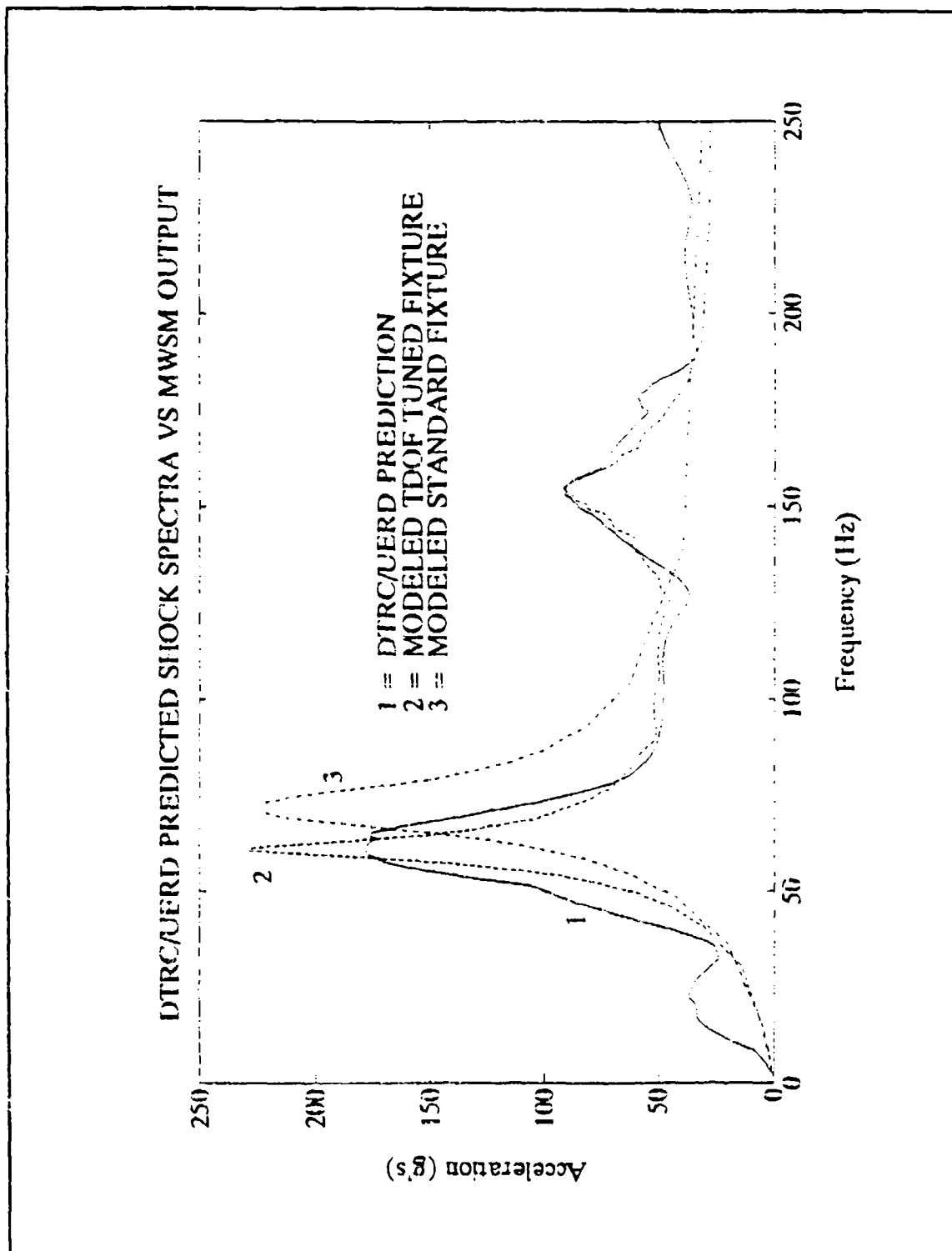


Figure 7. Comparison of the DTRC/UERD Predicted Vertical Acceleration Shock Spectra to the Modeled MWSM Fixtures for the Radar Receiver/Transmitter - 325 lbs

in this manner could be highly relied on to perform in a shipboard shock environment.

C. TWO DEGREE-OF-FREEDOM FIXTURE MODEL

1. Coupled and Uncoupled Natural Frequencies

The MWSM with equipment attached was modeled as two mass-spring-damper systems coupled together as shown in Figure 8. The anvil, when struck by the hammer, experiences a half-sine wave vertical acceleration impulse of approximately one millisecond in duration. The magnitude of the impulse is controlled by the hammer height prior to release. This impulse was modeled as shown in Figure 9.

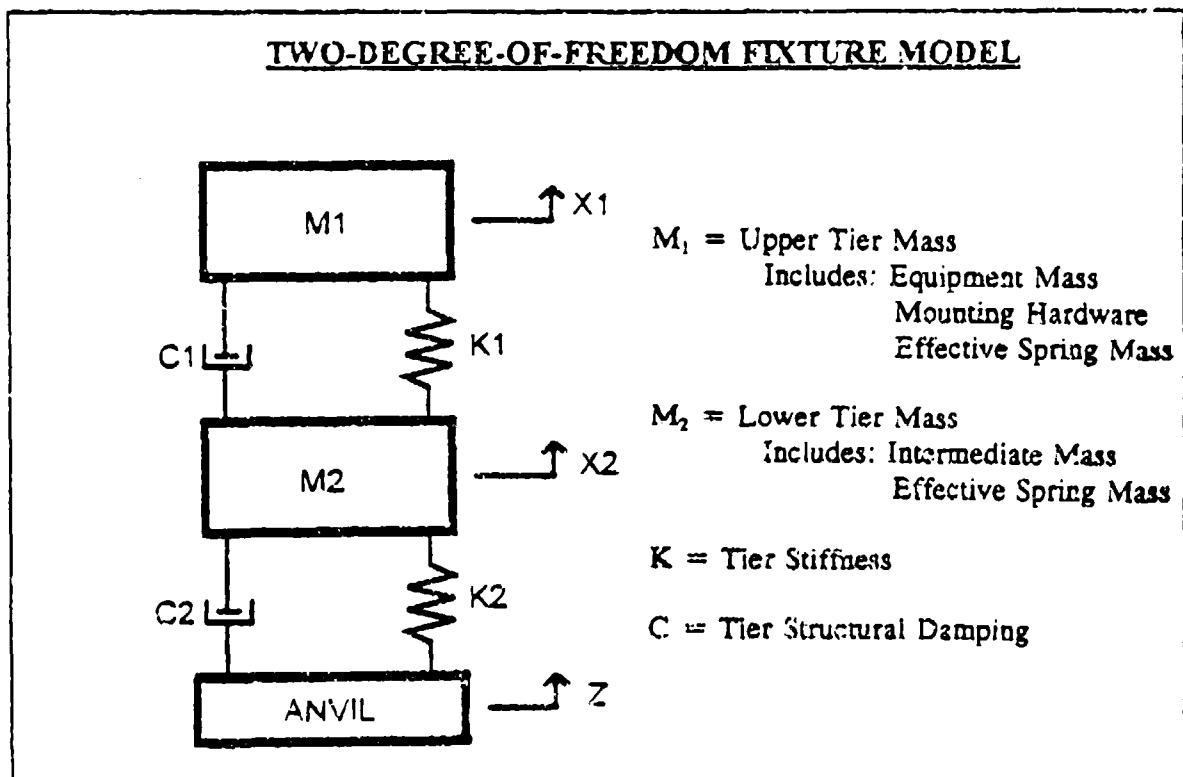


Figure 8. Modeled Two Degree of Freedom Fixture Subjected to Base Excitation.

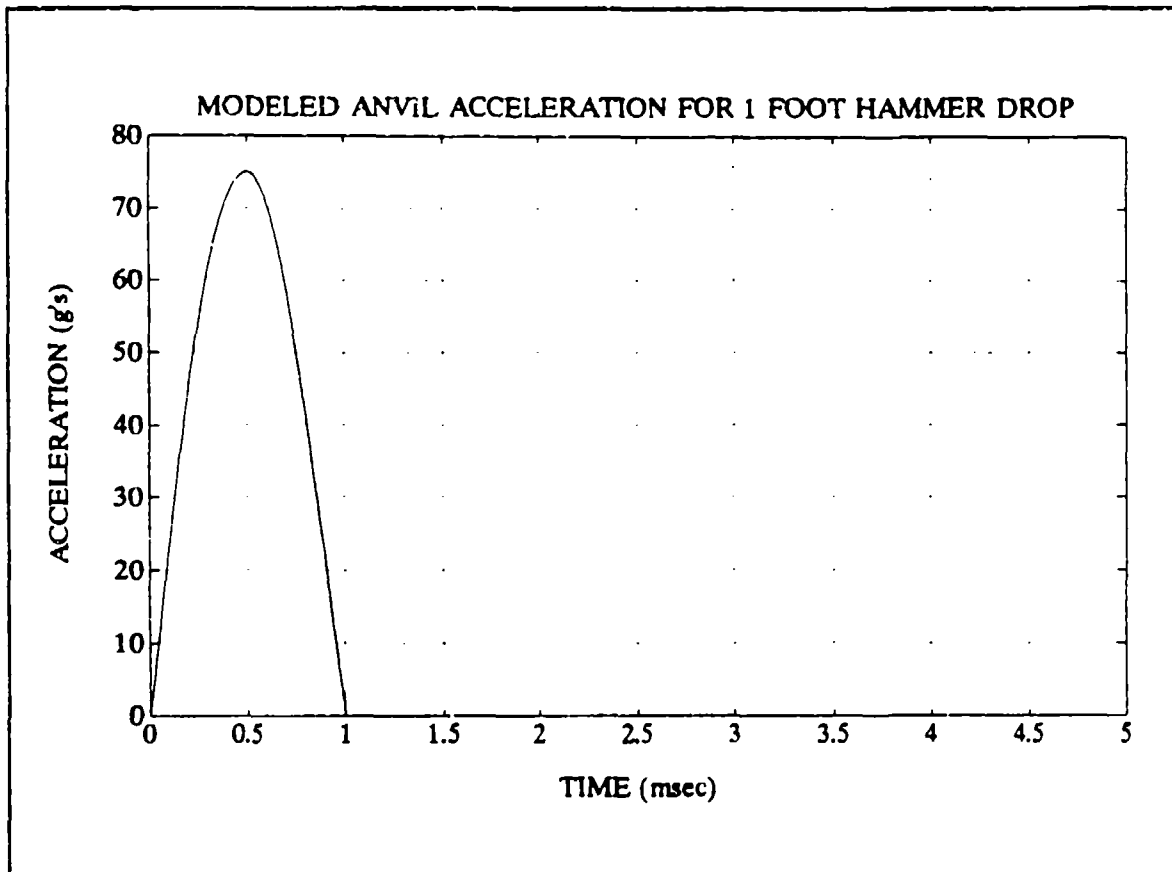


Figure 9. Modeled Anvil Table Acceleration for a One Foot Hammer Drop.

The mass M_1 represents the equipment mass and associated mounting hardware while M_2 represent the intermediate mass and support mountings. Each tier possesses characteristic stiffness and damping properties designated by K and C respectively.

To facilitate the solution of the mathematical model, the equations of motion for the system can be expressed in coordinates relative to the motion of the base, z . Expressing

the relative coordinates, y_1 and y_2 in terms of absolute coordinates,

$$\begin{aligned} y_1 &= x_1 - z \\ \dot{y}_1 &= \dot{x}_1 - \dot{z} \\ \ddot{y}_1 &= \ddot{x}_1 - \ddot{z} \end{aligned} \quad (1)$$

$$\begin{aligned} y_2 &= x_2 - z \\ \dot{y}_2 &= \dot{x}_2 - \dot{z} \\ \ddot{y}_2 &= \ddot{x}_2 - \ddot{z} \end{aligned} \quad (2)$$

yields the relative coordinate transformations for displacement, velocity and acceleration respectively. Solving for the coupled equations of motion gives the following matrix equation:

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{Bmatrix} \ddot{y}_1 \\ \ddot{y}_2 \end{Bmatrix} + \begin{bmatrix} c_1 & -c_1 \\ -c_1 & (c_1 + c_2) \end{bmatrix} \begin{Bmatrix} \dot{y}_1 \\ \dot{y}_2 \end{Bmatrix} + \begin{bmatrix} k_1 & -k_1 \\ -k_1 & (k_1 + k_2) \end{bmatrix} \begin{Bmatrix} y_1 \\ y_2 \end{Bmatrix} = \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{Bmatrix} -\ddot{z}_1 \\ -\ddot{z}_2 \end{Bmatrix} \quad (3)$$

By assuming the structural damping is small, the natural frequencies for free vibration can be found by assuming the damped and undamped natural frequencies are approximately equal. Neglecting the damping matrix:

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{Bmatrix} \ddot{y}_1 \\ \ddot{y}_2 \end{Bmatrix} + \begin{bmatrix} k_1 & -k_1 \\ -k_1 & (k_1 + k_2) \end{bmatrix} \begin{Bmatrix} y_1 \\ y_2 \end{Bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (4)$$

The system natural frequencies can then be found by solving the following determinate for the system eigenvalues:

$$\det|[K] - \omega_n^2[M]| = 0 \quad (5)$$

$$\text{where } \omega_n = 2\pi f_n$$

Defining the ratio between the upper and lower tier masses as,

$$\alpha = \frac{M_1}{M_2} \quad (6)$$

the relationship between the system's coupled and uncoupled natural frequencies¹ can be described by,

$$f_{n1} = \frac{1}{\sqrt{2}} \sqrt{f_1^2 + \alpha f_1^2 + f_2^2 - \sqrt{(f_1^2 + \alpha f_1^2 + f_2^2)^2 - (2f_1 f_2)^2}} \quad (7)$$

$$f_{n2} = \frac{1}{\sqrt{2}} \sqrt{f_1^2 + \alpha f_1^2 + f_2^2 + \sqrt{(f_1^2 + \alpha f_1^2 + f_2^2)^2 - (2f_1 f_2)^2}} \quad (8)$$

¹ These equations are based on the valid assumption that the structural damping is small and that the damped and undamped natural frequencies are equivalent.

where f_{n1} and f_{n2} are the coupled system natural frequencies and f_1 and f_2 are the uncoupled tier natural frequencies defined by:

$$f_1 = \frac{1}{2\pi} \sqrt{\frac{K_1}{M_1}} \quad (9)$$

$$f_2 = \frac{1}{2\pi} \sqrt{\frac{K_2}{M_2}} \quad (10)$$

If the desired system natural frequencies are known, equations (7) and (8) can be iteratively solved for the required uncoupled natural frequencies. Knowing the mass associated with each tier, equations (9) and (10) can be used to tune the fixture to the required stiffness, K .

From the shock spectrum shown in Figure 7, Corbell chose 60 Hz and 155 Hz as the system design resonant frequencies of the two DOF model. Figure 10 represents the iterative solution of equations (7) and (8) for the uncoupled natural frequencies in graphical form. As an example, the data was plotted as a function of the mass ratio $\alpha=1$ while fixing the lower tier natural frequency at $f_2=100$ Hz. Choosing an upper tier natural frequency of 94 Hz from the coordinate axis provides the desired system natural frequencies of 60 Hz and 155 Hz. This procedure was repeated for a mass ratio of 0.7 and 1.3 as shown in Figures 11 and 12.

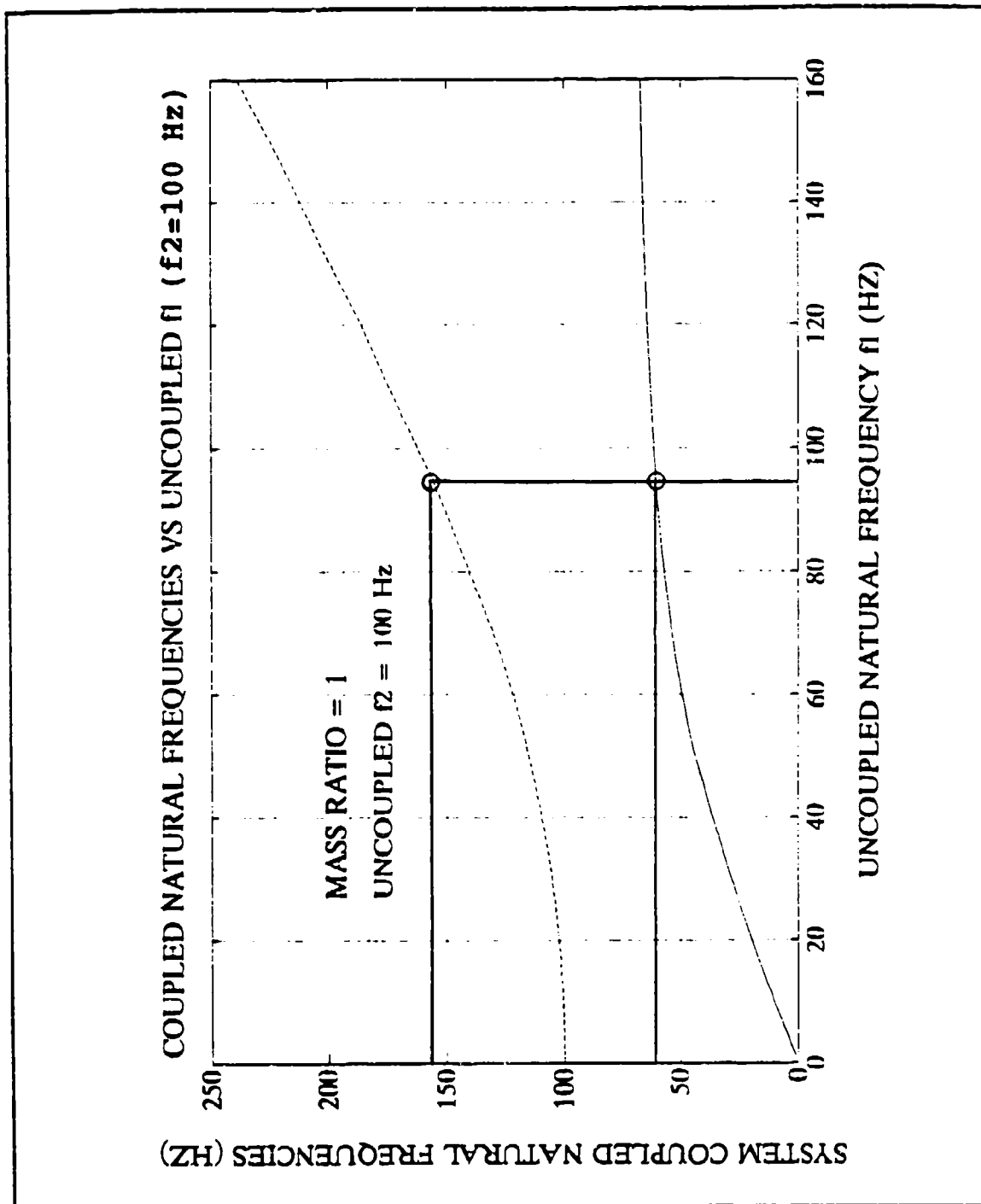


Figure 10. System Coupled Natural Frequencies as a Function of the Upper Tier Natural Frequency (f_1). Lower Tier Natural Frequency (f_2) Fixed at 100 Hz. Mass Ratio = 1.

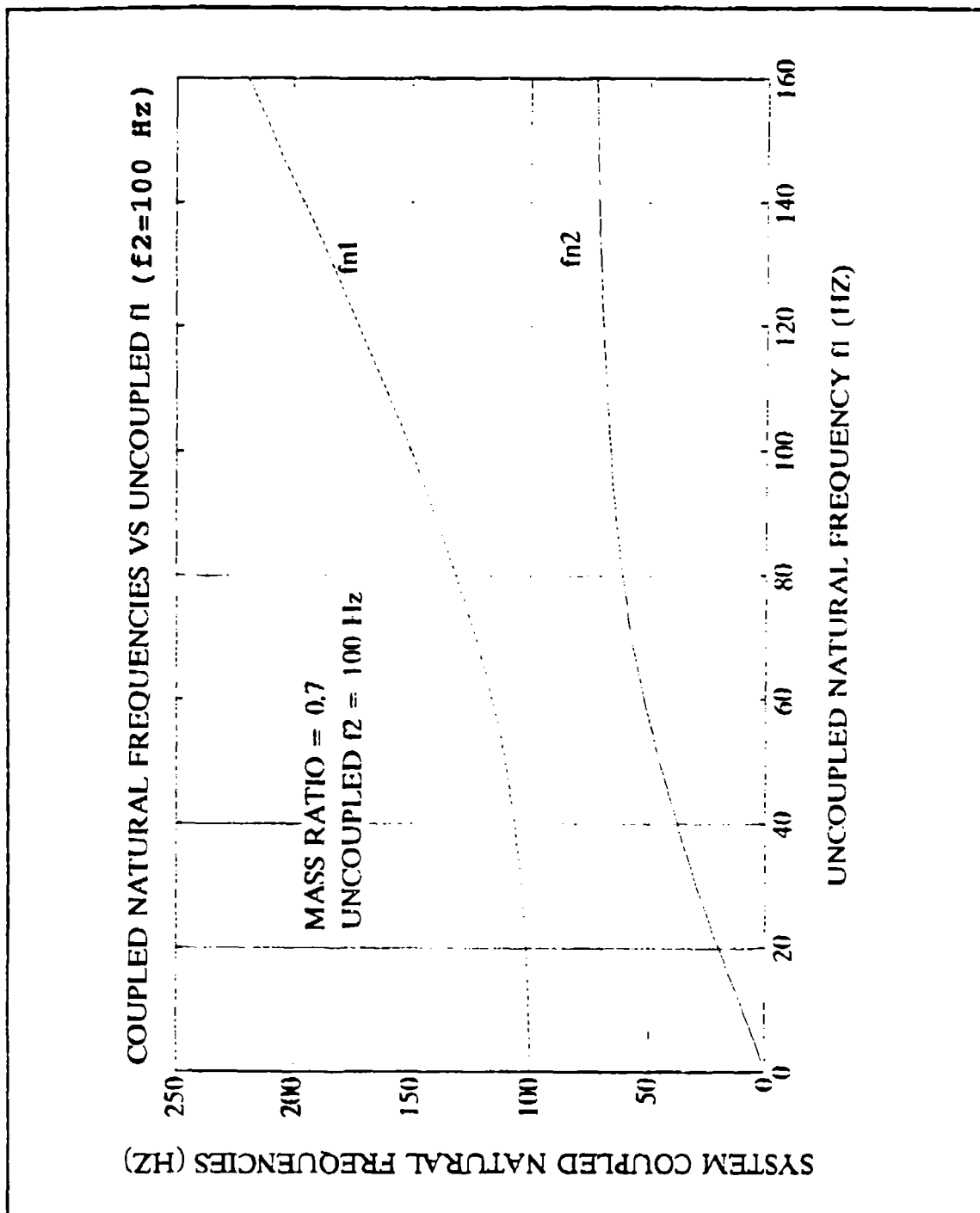


Figure 11. System Coupled Natural Frequencies as a Function of the Upper Tier Natural Frequency (f_1). Lower Tier Natural Frequency (f_2) Fixed at 100 Hz. Mass Ratio = 0.7.

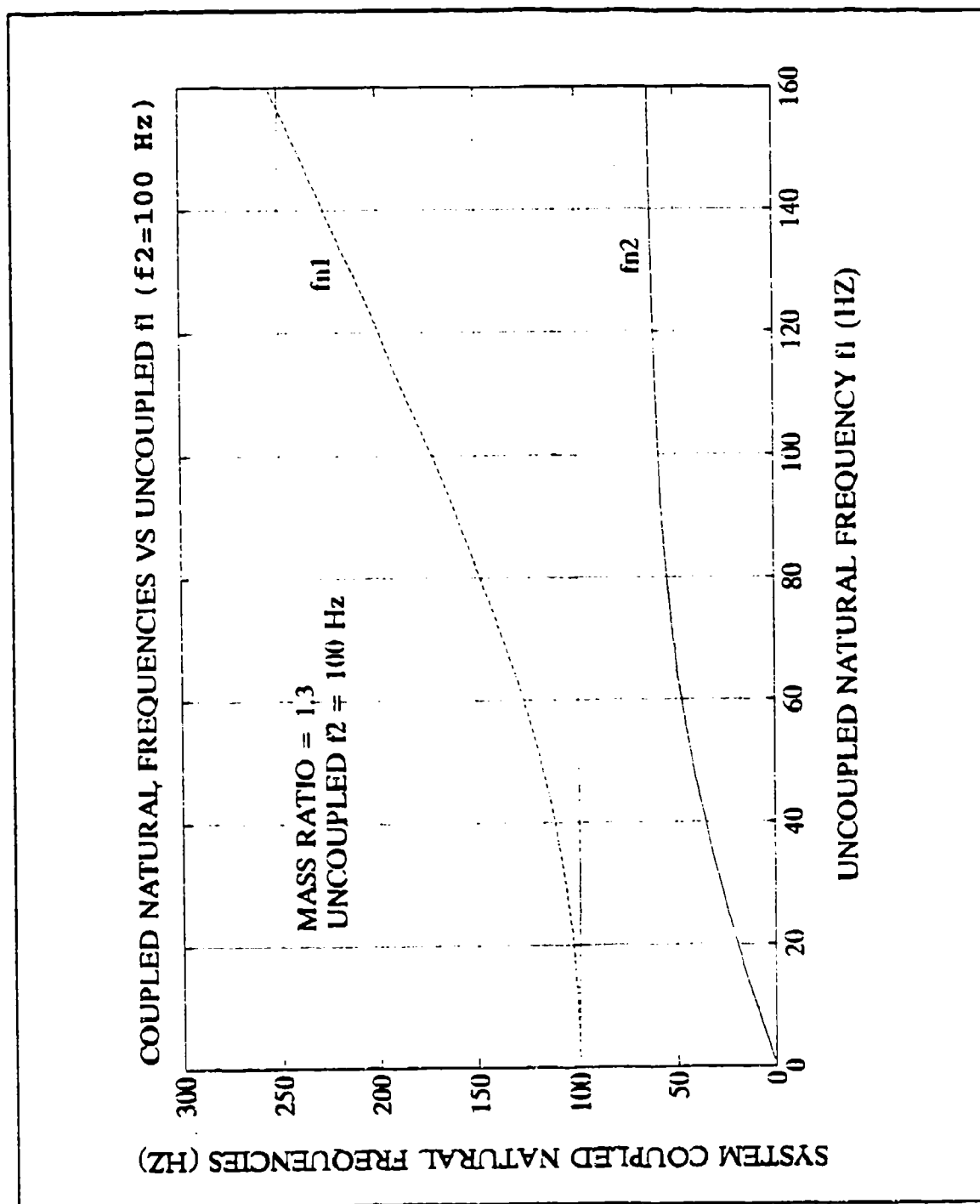


Figure 12. System Coupled Natural Frequencies as a Function of the Upper Tier Natural Frequency (f_1). Lower Tier Natural Frequency (f_2) Fixed at 100 Hz. Mass Ratio=1.3.

2. Analytical Response Calculations

To determine the response of the two DOF fixture to base excitation, the matrix equation 3 was first uncoupled using the normalized transformation matrix,

$$\begin{Bmatrix} y_1 \\ y_2 \end{Bmatrix} = \begin{bmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{bmatrix} \begin{Bmatrix} \eta_1 \\ \eta_2 \end{Bmatrix} \quad (11)$$

where the columns of $[\Phi]$ are the mode shape vectors (eigenvectors). The resulting uncoupled equations of motion in natural coordinates are:

$$\ddot{\eta}_1 + 2\zeta\omega_{n1}\dot{\eta}_1 + \omega_{n1}^2\eta_1 = -\ddot{Z}_1 \quad (12)$$

$$\ddot{\eta}_2 + 2\zeta\omega_{n2}\dot{\eta}_2 + \omega_{n2}^2\eta_2 = -\ddot{Z}_2 \quad (13)$$

where ζ is the critical damping ratio estimated from the MWSM calibration test data compiled by Costanzo and Clements (1988). Examination of the calibration data showed that the test weight acceleration response damped out in approximately .3 to .5 seconds after hammer impact. This corresponds to a critical damping ratio of between three to five percent. Figures 13 through 15 show the damped acceleration response for the two DOF model for three values of the critical damping ratio.

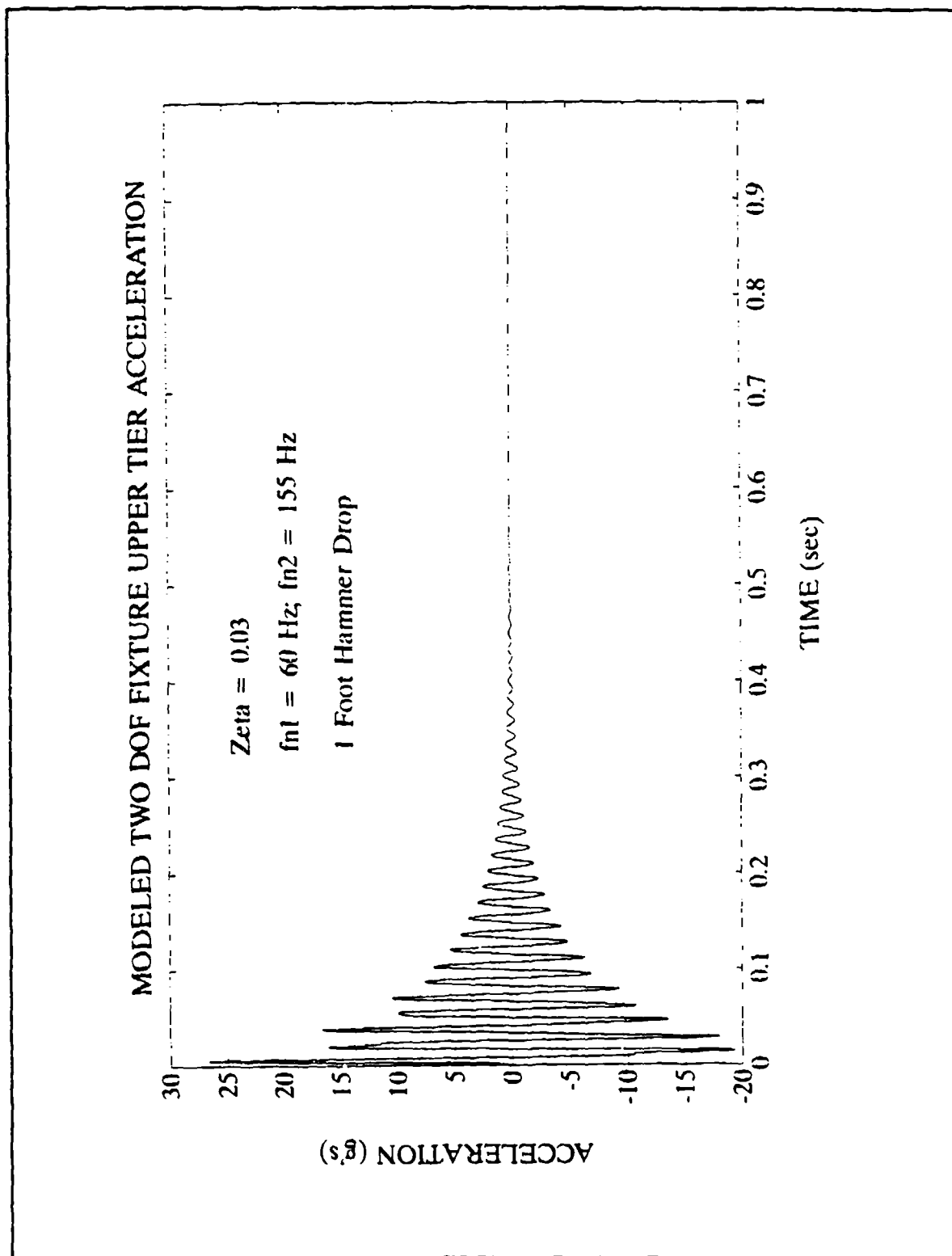


Figure 13. Modeled Two DOF Fixture Acceleration Response for Zeta = 0.03.

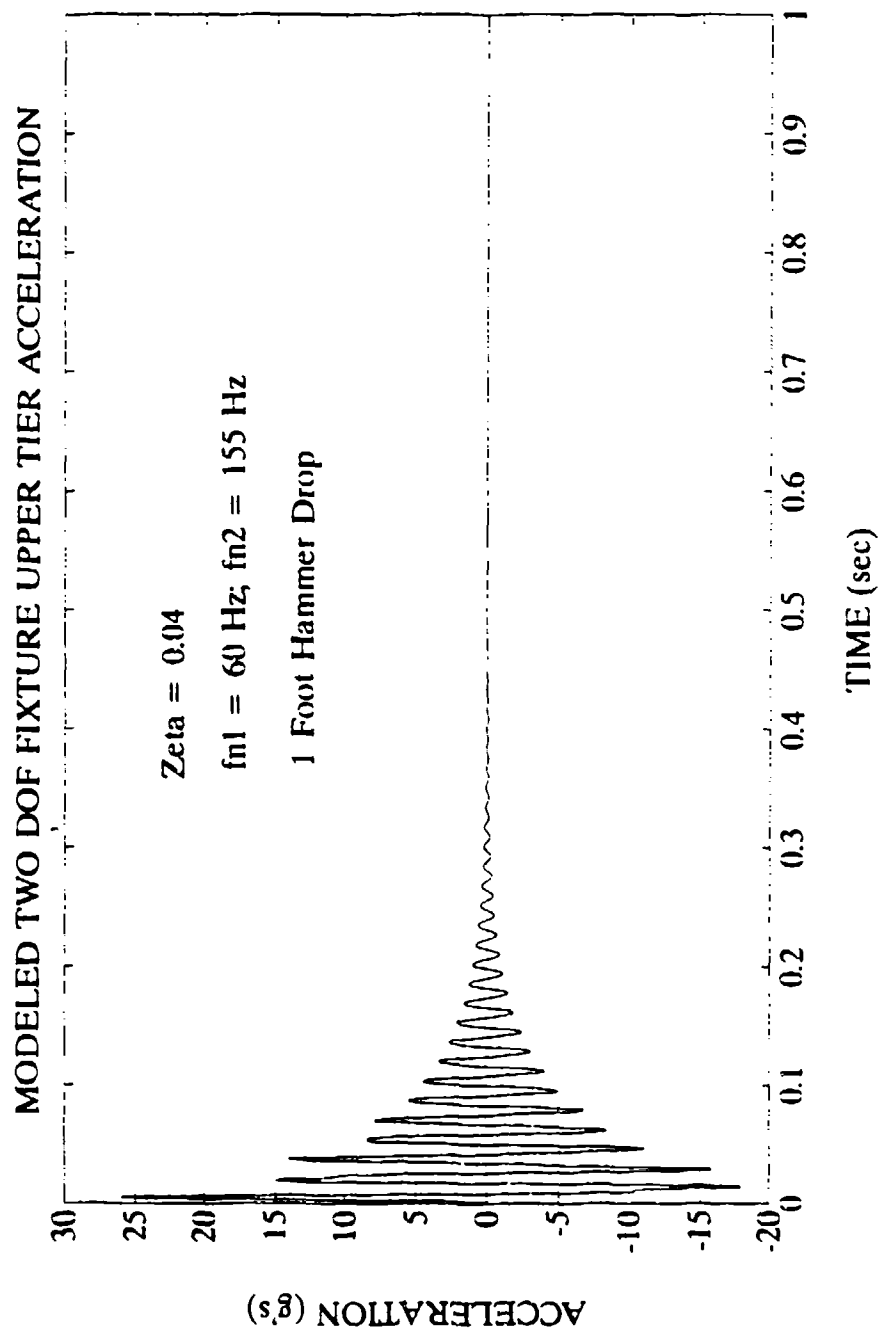


Figure 14. Modeled Two DOF Fixture Acceleration Response for Zeta = 0.04.

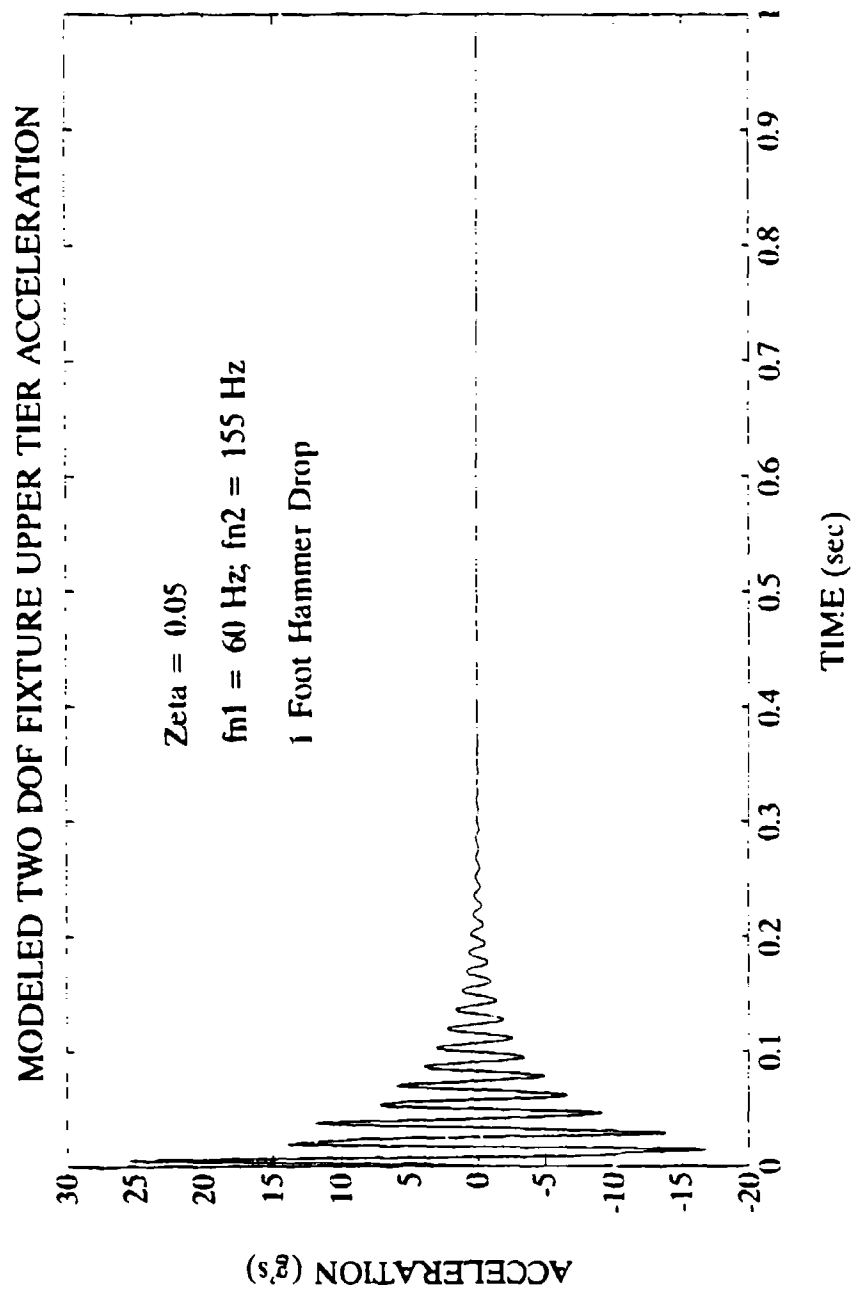


Figure 15. Modeled Two DOF Fixture Acceleration Response for Zeta = 0.05.

Equations (12) and (13) were solved independently using a continuous-time linear system simulation. The solution in natural coordinates was then transformed into relative coordinates using equation (11) and then to absolute coordinates using equation (1) and (2) to obtain the system displacement, velocity and acceleration responses. The MATLAB™ algorithm solving the two DOF system is presented in Appendix A.

The two DOF model and numerical response algorithm provided the prerequisite information needed for designing and predicting the response of the proposed fixture. In Chapter IV, a semi-definite three degree of freedom model will be introduced which accounts for the unconstrained 4500 lb anvil table interaction with the fixture after the time of hammer impact.

III. TWO DEGREE-OF-FREEDOM TUNED FIXTURE DESIGN

The designs discussed in this chapter are based on the two DOF model presented in Chapter II. The procedure followed was to first identify the desired system natural frequencies for the fixture and then determine the required tier natural frequencies using equations (7) and (8). Once the tier natural frequencies were known, the required stiffness for each tier could be determined using equations (9) and (10). The mass, stiffness and damping for each tier were input into a numerical algorithm to determine the fixture response to the half-sine wave impulse shown in Figure 9. The displacements of each tier relative to the anvil table were determined for a variety of hammer heights, equipment weights and system natural frequencies. The maximum dynamic relative displacement within the fixture was obtained for the above conditions and then used to conduct a static stress analysis of the fixture. A detailed procedure of the design process is presented below.

A. DESIGN OF THE TWO DOF TUNED FIXTURE

1. Design Considerations

The design phase of the research began with identifying the two resonant frequencies desired for the fixture. After referring with Mark McClean at Naval Sea Systems Command, the previously desired frequencies of 60 Hz

and 155 Hz had been lowered to 30 Hz and 80 Hz for the goal frequencies. Modal testing in the applicable areas of the ship indicated that the shipboard frequencies were lower than the DTRC/UERD predictions.

A second design consideration was cost. The Navy could easily develop a pulse-shape matching machine capable of reproducing shipboard shock spectra but the cost per machine would be significant. The Navy currently has 13 MWSM in use and replacing them with million dollar machines is not economically feasible. The design approach was to develop a fixture that used many of the current parts and accessories already in use on the MWSM. By doing so, fabrication and material cost could be minimized.

MIL-S-901D provides some general guidelines for designers building equipment to endure shock excitations, many of which are applicable to the fixture design. Materials recommended for structural members are those with high yield strength, high ductility, high fracture toughness and when possible, light weight. Cast iron and cast aluminum have generally proven to be unsatisfactory when used as strength members due to their high notch sensitivity and brittleness. Areas normally subjected to compressive stress under static loading conditions may experience tensile stress when subjective to the cyclic loading experienced under shock and vibration. This fact must be kept in mind specifically when designing welded and bolted joints. These joints tend to cause

stress concentrations which may be acceptable in compression but may lead to failure in tension.

2. Preliminary Design

The following designs were based on the two degree of freedom model shown in Figure 8. The focus here was to define a structure with a suitable set of tier stiffness and mass components which would provide the desired response without yielding or structural failure. Numerous designs were explored and then narrowed to two for this research.

a. Coil Spring Deck Design

The soft deck simulator shown in Figure 3 provides some obvious qualities which would be quite desirable for this design. The most significant quality being the ease in tuning the fixture. As the weight of the attached equipment varies the fixture is easily tuned by adding or removing spring cartridges to maintain the desired natural frequency. A second quality is the relatively large vertical displacement allowed within the fixture. Since the relative displacement is inversely proportional to square of the natural frequency, it becomes increasingly necessary to allow more relative motion between the tiers as lower frequencies are trying to be obtained. The coil springs will allow up to two inches of travel without being over stressed or becoming coil bound.

Another feature of using coil springs is the ability to distribute the spring force over the entire surface

area of the table top which has two advantages. First, as mentioned in Clements (1972), the MWSM is not a perfect machine and does not always produce a strictly vertical impulse on the anvil table. By distributing the spring force, the fixture becomes more stable since it can better resist the rotational moment produced by the supported mass when its center of gravity is not in line with the direction of the impulse force. Secondly, since the force is distributed, the bending moments applied to the rigid intermediate mass (M_2) are minimized. This reduces unwanted secondary frequencies generated by deflections within the structure of the intermediate mass.

To achieve the desired system natural frequencies of 30 Hz and 80 Hz the required uncoupled frequencies from equations (7) and (8) were determined to be $f_1=48$ Hz and $f_2=50$ Hz. The mass ratio α was set equal to one with each tier mass weighing 3000 lbs. This would leave 1400 lbs available of the 7400 lbs allowable on the MWSM for mounting hardware and the inclining fixture called for in MIL-S-901D. Using equations (9) and (10) the required upper and lower tier stiffness was determined to be $K_1=7.66 \times 10^5$ lb/in and $K_2=7.06 \times 10^5$ lb/in.

The coil springs currently used in the soft deck simulator have a stiffness² of 2205 lb/in and weigh

² This is the mean stiffness per coil spring as determined by Steve Schecter at Hughes Air Craft, Fullerton, Ca.

approximately 15.5 lbs each. This would require 348 springs on the upper tier and 320 springs on the lower tier for a combined weight of 10,354 lbs for the springs alone. This is obviously an unacceptable weight for use on the MWSM.

An alternate off-the-shelf spring was selected with a stiffness of 6090 lb/in, a weight of 18 lb per spring and similar dimensions. This would require 126 springs on the upper tier and 116 springs on the lower tier for a total weight of 3280 lbs. This reduced the spring weight significantly but still consumed too much of the weight capacity of the MWSM. The advantages of the coil spring design had to be abandoned for a design with a higher stiffness-to-weight spring mechanism. The beam loading concept used in the standard fixture was a logical choice.

b. Beam Spring Design

- Obtaining the necessary stiffness while minimizing weight encouraged the use of beams for the spring elements. The beam elements were modeled as being simply supported with symmetric loading as shown in Figure 16. The stiffness of the beam is given by equation (14).

$$K = \frac{F}{y} = \frac{6EI}{a^2(3L-4a)} \quad (14)$$

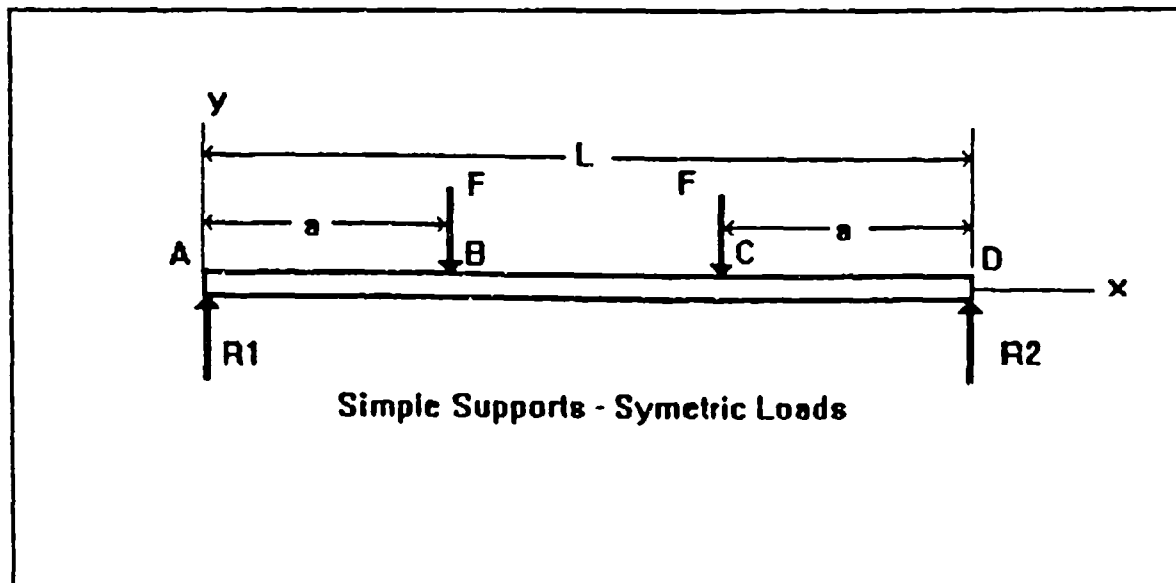


Figure 16. Beam Bending Model for Determining Beam Stiffness.

This design was based on using the existing shipbuilding rails as the spring beam supports. From the drawings of the MWSM in MIL-S-901D, this fixed L at approximately 50 inches. The load spacing was set to 24 inches making $a=13$ inches. Young's Modules was taken to be $E=30 \times 10^6$ psi for steel and the area moment of inertia was set to 9.2 in^4 , based on the current back-to-back C channels (4x7.25#) being used on the standard fixture. The result was a stiffness of $2.17 \times 10^5 \text{ lb/in}$ with each beam set weighing approximately 70 lb. To achieve the desired natural frequencies, the lower tier required 4 beam sets and the upper tier required 3 beam sets for a total weight of 490 lbs. The beams provided a significant weight savings and became the focus of the detailed design.

B. DETAIL DESIGN OF THE BEAM SPRING FIXTURE

In designing the two DOF fixture, priority was placed on the ability to tune the fixture to achieve the desired system natural frequencies over the widest possible range of equipment weights. An equally important consideration was minimizing the weight of the fixture while still maintaining the characteristics of the shipboard shock wave. A 1/4 scale model of the proposed two DOF fixture is presented in Figure 17. The following paragraphs highlight the significant factors dealt with during the design process.

1. Choice of Beams

The back-to-back C channels used on the standard fixture were first investigated for use in the two DOF design. These channels would mount to the shipbuilding support rails in the current fashion using the existing clamping method. However, in an effort to simplify the design and to use a symmetric cross section, I-beams were ultimately chosen. The I-beams used in the design are W4x13 beams with an area moment of inertia of 11.3 in^4 resulting in a stiffness of $2.46 \times 10^5 \text{ lb/in}$. By modifying the existing clamping design, the fixture could be assembled without requiring any drilling or welding of the I beams in areas of high stress. To achieve the system natural frequencies desired, required four I-beams on the bottom tier and three on the top tier. To add more flexibility in mounting the equipment to the fixture, the design allows



Figure 17. 1/4 Scale Model of the Proposed Two DOF Fixture for the MWSM.

for interchanging the I-beams with the back-to-back C channels on the upper tier.

2. Tuning The Fixture

As the equipment weight attached to the fixture changes, the fixture is tuned by altering the stiffness-to-mass ratio of the upper and lower tiers. On the upper tier, the stiffness is increased by reducing the dimension "a" shown in Figure 16. The initial design of the lower tier allowed for the same adjustment to alter the stiffness. In an effort to increase the rigidity of the intermediate mass M_2 , and to increase stability by lowering the center of gravity of the entire fixture, the design was changed fixing $a=13.5$ inches for the lower tier beams. The lower tier natural frequency is adjusted by adding or removing ballast to the intermediate mass as necessary.

When tuning the fixture, the effective mass of the springs was taken into account. Using Furtis (1972) as a reference, the effective mass of the spring elements was determined to be 52% of the total beam mass. For K_2 , this mass was added to the intermediate mass M_2 . For K_1 , 52% of the spring mass was added to the upper tier mass M_1 and the remaining 48% added to M_2 since it would be in motion with the intermediate mass. The distribution of the effective spring mass is some what trivial since its magnitude is quite small relative to the tier masses.

3. Intermediate Mass (M_2) Design

As shown earlier in equations (7) and (8), the ratio of the upper tier mass to lower tier mass (α) has an effect on the system response to the uncoupled tier frequencies. The plot in Figure 18 depicts this relationship. For desired system natural frequencies of 30 Hz and 80 Hz, the plot shows the required tier natural frequencies as a function of α . As α increases, the required upper tier natural frequency decreases while the lower tier increases. For α greater than 1.3, the required uncoupled natural frequencies diverge very quickly indicating that the desired system response is not achievable. This signifies that the intermediate mass, M_2 , must be greater than or equal to 77% of the equipment mass. The design weight of M_2 including the mounting hardware is 1700 lbs and will require adding ballast for equipment weights over 2200 lbs. The result is, M_2 will consume a significant portion of the weight capacity of the MWSM and will limit equipment weight to approximately 3500 lbs. This weight allowance, although somewhat reduced, will still allow significant numbers of equipments to be qualified with this design.

A second design consideration for M_2 was that it must be stiff enough to appear as a rigid body to the rest of the fixture. This would prevent unwanted deflections and vibrations not associated with the desired system response. The necessary stiffness was achieved by forming a grid

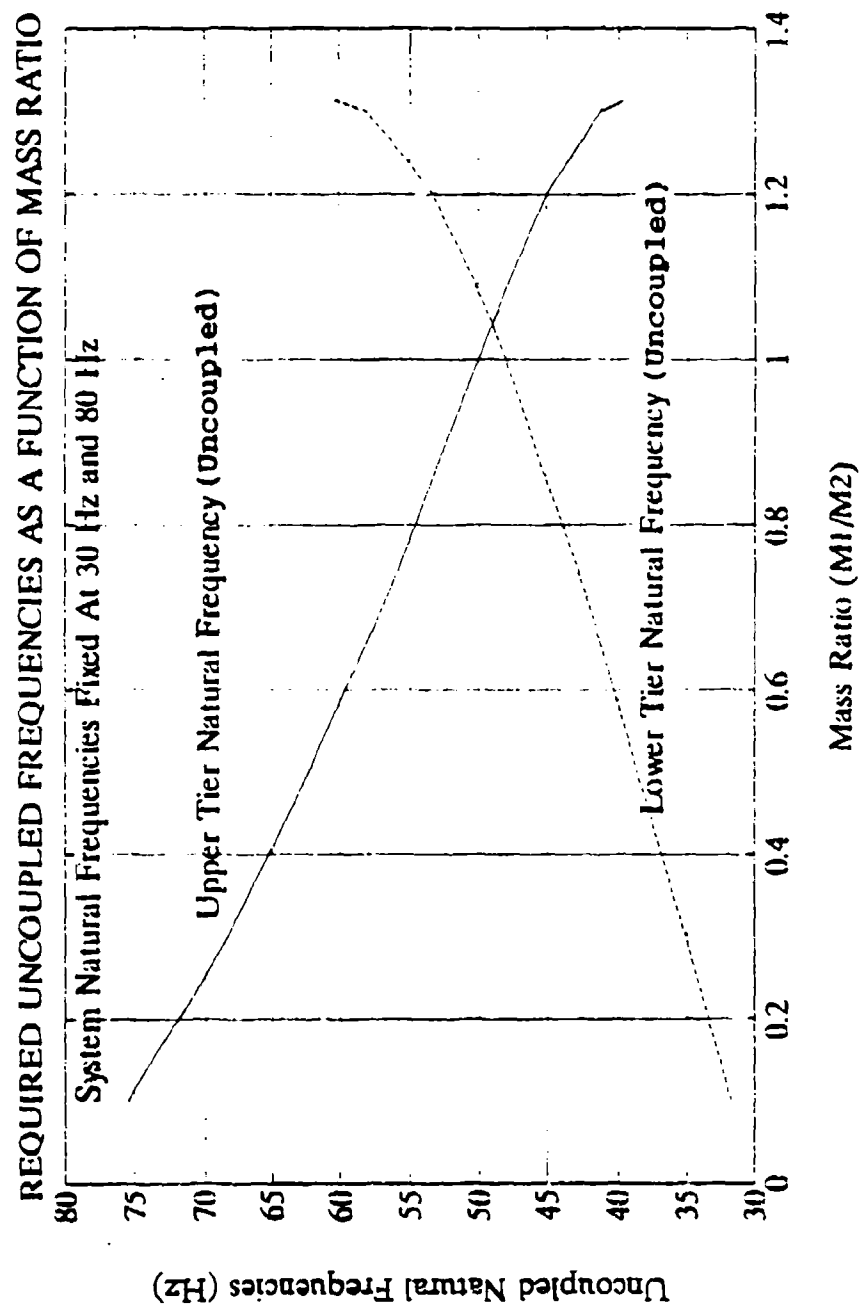


Figure 18. Required Upper and Lower Tier Natural Frequencies as a Function of the Mass Ratio.

structure of I-beams with an overall stiffness of 1.0×10^7 lb/in which is approximately 13 times higher than K_1 or K_2 .

To preclude the need for fabricating a different spring beam for the upper tier and lower tier, the intermediate mass M_2 provides the same mounting geometry as the shipbuilding support rails mounted on the MWSM anvil table. This allows interchanging the upper and lower tier springs as well as the back-to-back C channels currently used as part of the standard fixture. In some instances, mounting equipment on the upper tier with back-to-back C channels will be more convenient and provide more flexibility in tuning the fixture.

C. STRESS ANALYSIS OF THE TWO DOF FIXTURE

Considering only deflections in the vertical direction, evaluation of the most likely modes of failure focused on the tensile stress in the fasteners and bending stress in the spring beams and clamps. The numeric model was loaded to its weight capacity and then excited with the maximum expected acceleration level. The upper tier weight was set to 3500 lbs and the lower tier weight set to 2700 lbs. Using the 300 Hz filtered data shown in Figure 19, the maximum expected anvil acceleration was picked to be 200 g's for a five foot hammer drop. Using these parameters in the numeric model, the relative displacements of the upper and lower tiers with respect to the anvil table were determined. Figure 20 is a

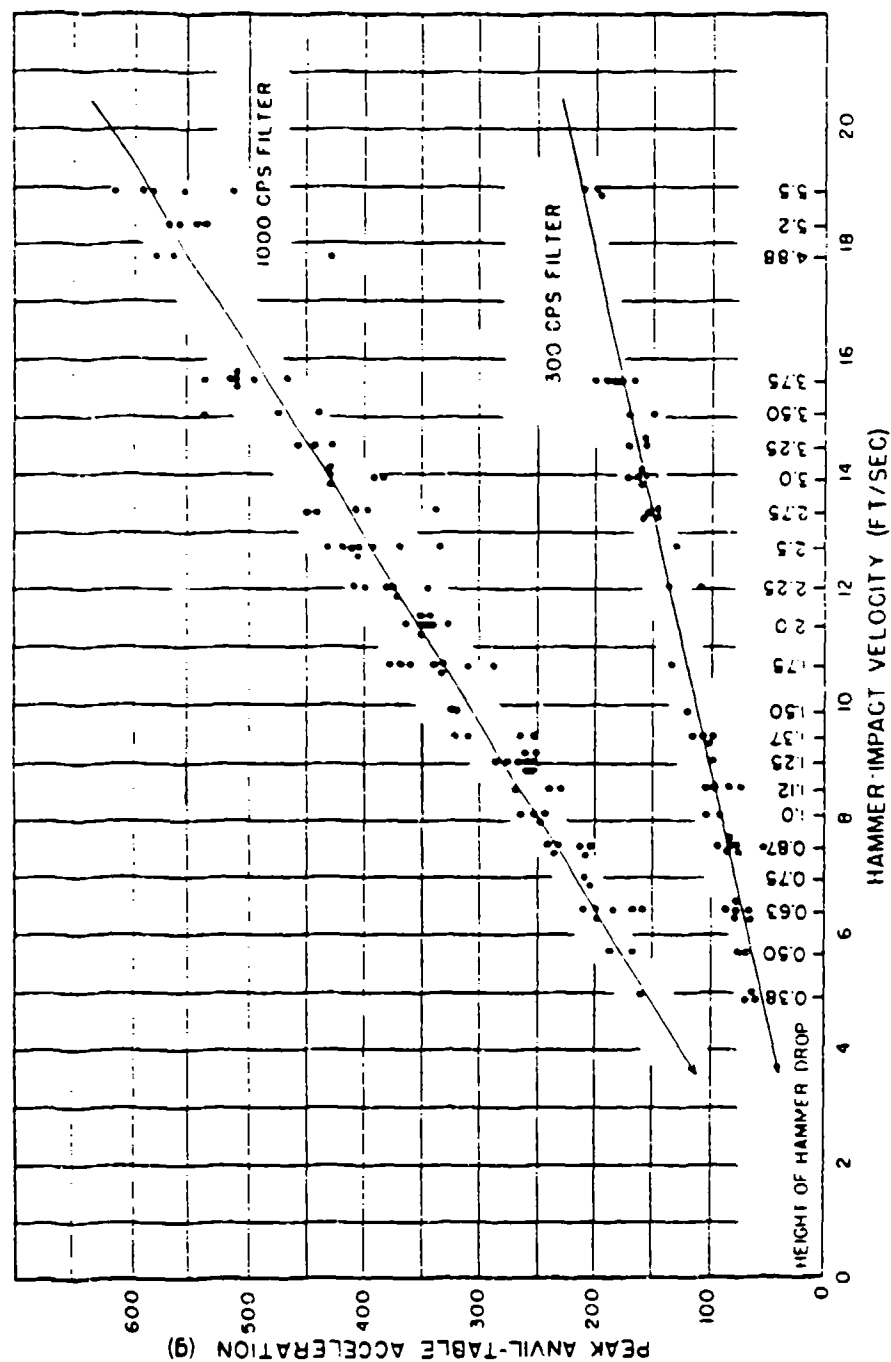


Figure 19. MWSM Peak Anvil Table Acceleration vs. Hammer Height. Courtesy of Clemenets (1972).

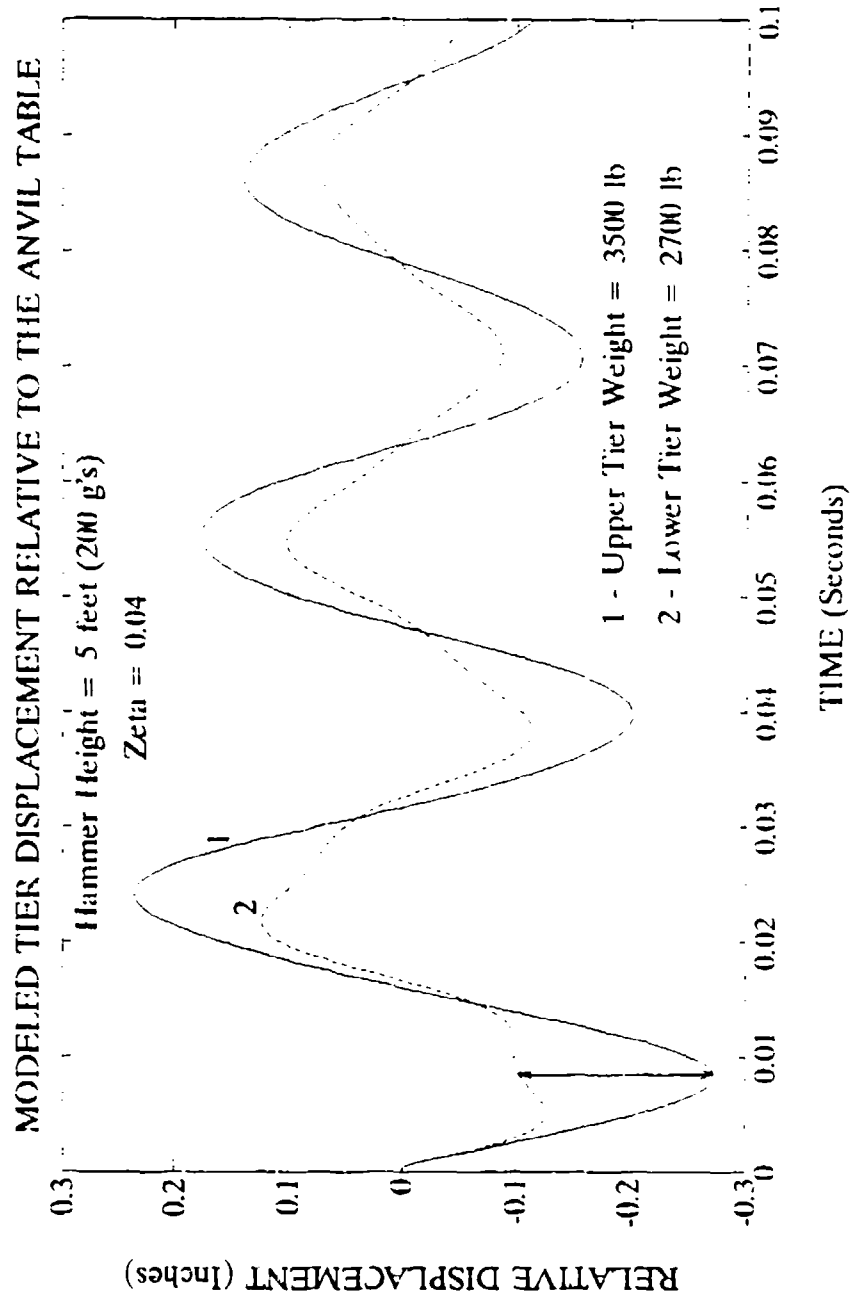


Figure 20. Displacements of the Upper and Lower Tier Relative to the Anvil Table.

time history plot of these displacements. The maximum relative displacement of 0.18 inches occurs at $t=.009$ seconds between the upper and lower tiers. This displacement, y , is used in equation (15) to determine the equivalent static force, F . Referring to Figure 16:

$$F = \frac{6yEI}{a^2(3L-4a)} \quad (15)$$

Then from equation (16), the maximum bending moment in the spring beam can be calculated.

$$M = Fa \quad (16)$$

Knowing the bending moment, M , the maximum bending stress was determined from equation (17);

$$\sigma_{\text{bending}} = \frac{Mc}{I} \quad (17)$$

where $c=2.0$ inches for the four inch I-beam. The shear stress in the beams was determined to be negligible and not included in the calculations. (Shear stress is zero at the extreme fiber where the bending stress is maximum).

The equivalent static force ,F, was used correspondingly to determine the stresses in the clamps and bolts fastening the fixture together. The maximum stress in the spring beams was determined to be 47 kpsi and 13 kpsi in the clamps. Maximum tensile stress in the bolts was determined to be 7.5 kpsi.

D. DESIGN DRAWINGS AND MATERIAL SELECTION

Detailed design CAD drawings of the proposed two DOF tuned fixture were provided to Naval Sea Systems Command for approval and fabrication. A set of these drawings have been included in Appendix B of this document.

The material selected for the spring beams has a yield strength of 50 kpsi. Materials selected for all other structural elements is standard 36 kpsi yield strength mild steel. Bolt material is high quality grade 5 carbon steel alloy. All materials have been received and fabrication expected to begin by 1 June 1993. Upon completion, the fixture will be sent to Naval Underwater Systems Center, New London, Ct. for testing.

E. PREDICTED RESPONSE FOR THE TWO DOF FIXTURE

To predict the performance of the proposed two DOF fixture, a sensitivity analysis of the model was performed to determine the tunability and the response characteristics of the fixture as equipment weight was varied. For the analysis,

the desired system natural frequencies were chosen to be 20 to 40 Hz for f_{n1} and 70 to 90 Hz for f_{n2} . The equipment weight including the mounting hardware varied from 500 lb to 3500 lb.

1. Tunability of the Fixture

To examine the tunability of the fixture, a worksheet was produced in Mathcad® which iteratively solves equations (7) and (8) for the required uncoupled tier frequencies f_1 and f_2 given the desired system natural frequencies f_{n1} and f_{n2} and the mass ratio α . Using the worksheet, the required tuning can easily be determined as equipment weight or desired system natural frequencies change. A copy of this worksheet is provided in Appendix C. Figure 21 shows the achievable system

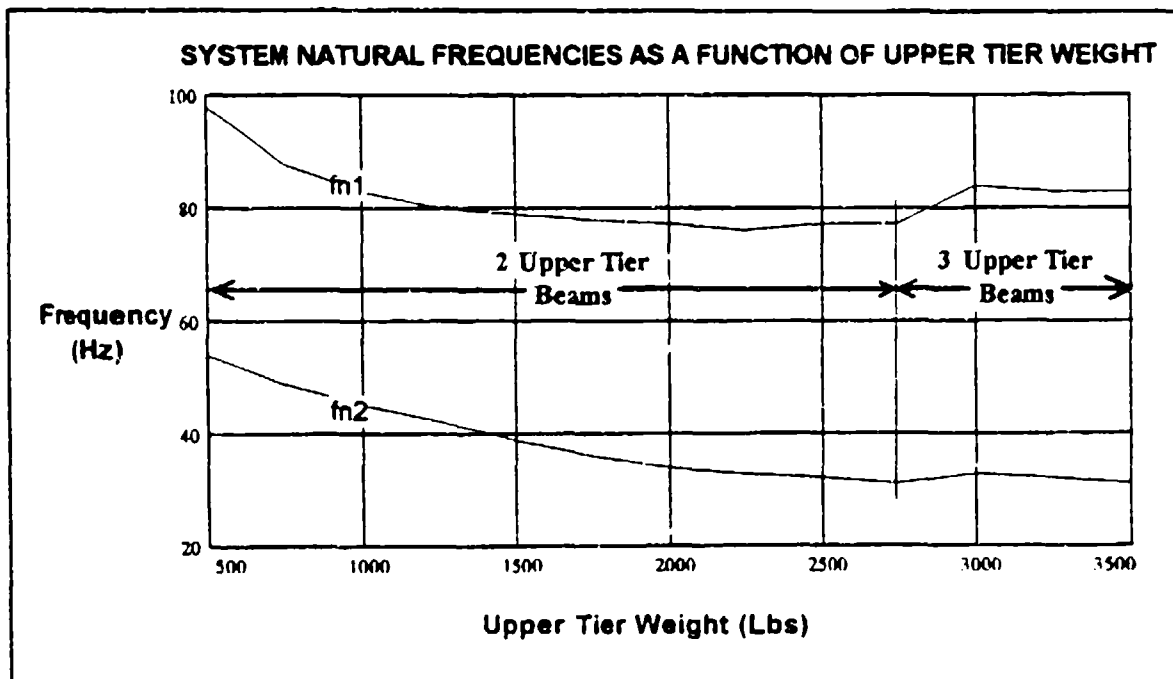


Figure 21. System Natural Frequencies as a Function of the Upper Tier Weight

TABLE I. TUNING FOR VARIOUS UPPER TIER WEIGHTS.

Upper Tier Weight ¹ (lb)	Lower Tier Weight (lb)	"a" Upper Tier ² (inches)	# Beams Upper Tier	fn ₁ (Hz)	fn ₂ (Hz)
500	2500	16	2	54	98
750	2500	16	2	49	88
1000	2500	16	2	45	83
1250	2500	16	2	42	80
1500	2500	16	2	38	79
1750	2500	16	2	36	78
2000	2500	16	2	34	77
2250	2500	16	2	32	76
2500	2500	15	2	32	77
2750	2500	15	2	30	77
3000	2700	14	3	33	84
3250	2700	14	3	32	83
3500	2700	14	3	31	83

1-Upper tier weight includes equipment, mounting hardware and effective spring weight.

2-Upper tier equipment mounting spacing. Refer to figure 16.

natural frequencies as the weight of the upper tier is varied between 500 and 3500 lbs. TABLE I shows the necessary adjustments which would be required to tune the fixture. The sharp rise in system natural frequencies for upper tier weights less than 1500 lbs indicates that adding ballast to the upper tier may be required for some light weight equipment.

A second analysis was performed to determine the range of system natural frequencies obtainable for a specified equipment weight. The equipment weight was given to be 2500 lbs and the fixture tuned by varying the weight of M_2 and adjusting the equipment mounting spacing ("a" in Figure 16). For an equipment weight of 2500 lbs, Figure 22 demonstrates the obtainable system natural frequencies. The data provided in TABLE II indicates the necessary adjustments. Figures 21 and 22 demonstrate that this fixture design could be tuned to the desired system natural frequencies under a variety of loading conditions.

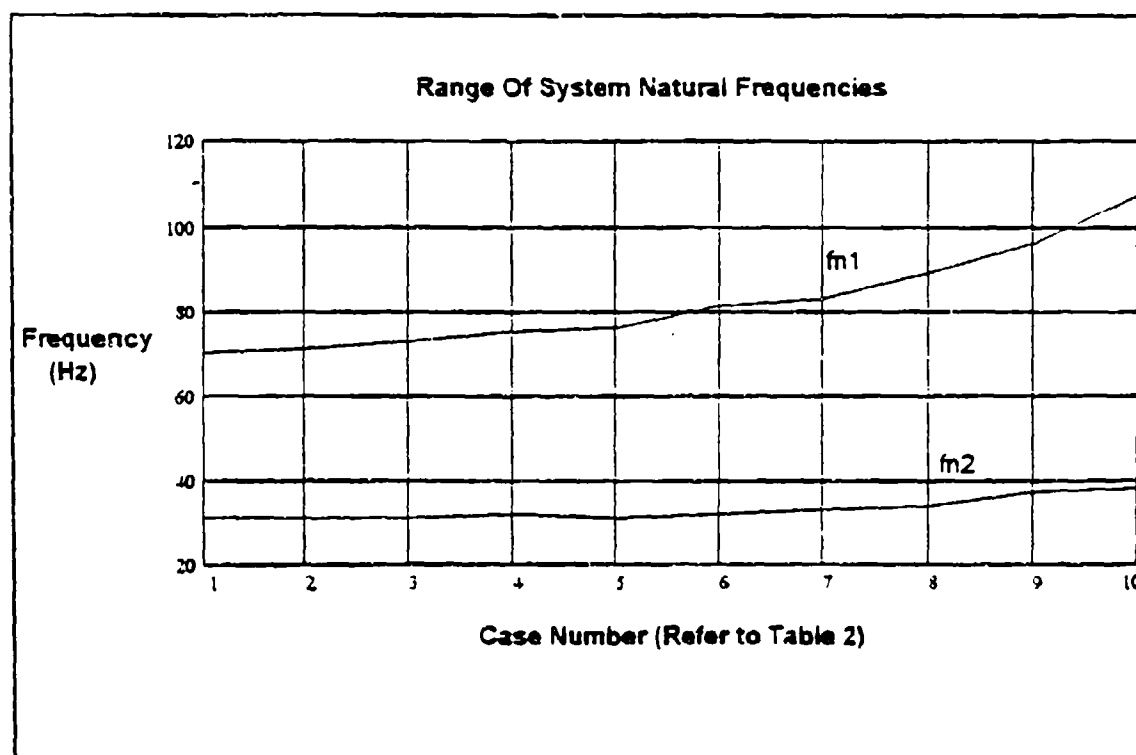


Figure 22. Range of System Natural Frequencies for a 2500 lb Upper Tier Weight.

TABLE II. TUNING FOR AN UPPER TIER WEIGHT OF 2500 LBS.

Case Number	Lower Tier Weight (lb)	"a" Upper Tier (inches)	# Beams Upper Tier	fn_1 (Hz)	fn_2 (Hz)
1	3000	16	2	31	70
2	3000	15	2	31	71
3	2750	16	2	31	73
4	2750	15	2	32	75
5	2500	16	2	31	76
6	2250	15	2	32	81
7	2250	14	2	33	83
8	2000	13	2	34	89
9	2000	14	3	37	96
10	1750	13	3	38	107

2. Two DOF Fixture Response Characteristics

As mentioned in Chapter II, for the fixture to be effective, it should provide the equipment with accelerations that are "characteristic" of the expected shipboard excitations. Since the desired frequencies have been lowered by NSWC to 30 Hz and 80 Hz, it is difficult to compare the modeled results to the DTRC/UERD predictions. However, as shown for five different equipment weights, Figures 23 through 28 demonstrate that the modeled time history accelerations and associated shock spectra provide significant amplitudes of acceleration at the desired frequencies.

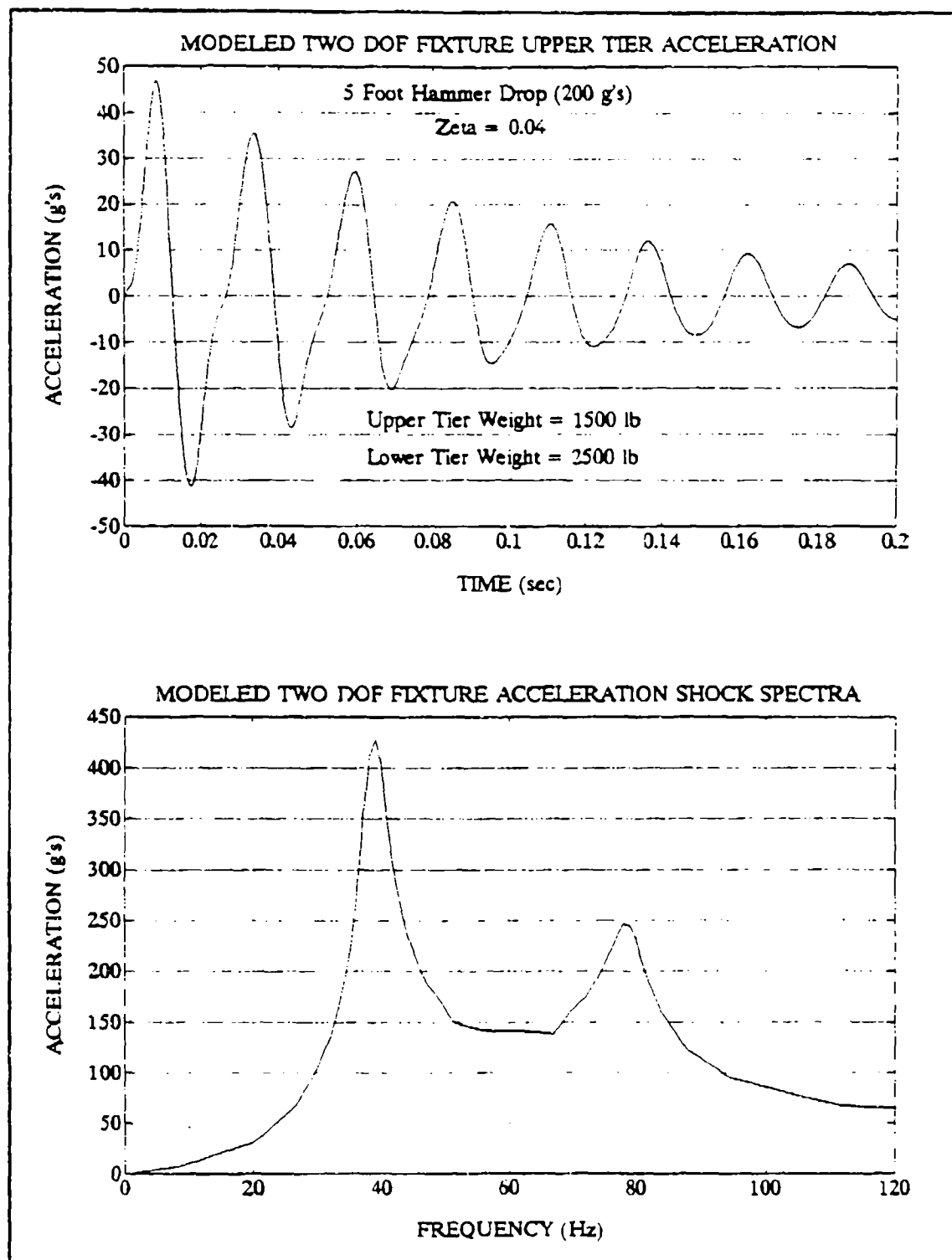


Figure 23. Predicted Acceleration Wave Form and Shock Spectrum for 1500 lb Upper Tier Weight.

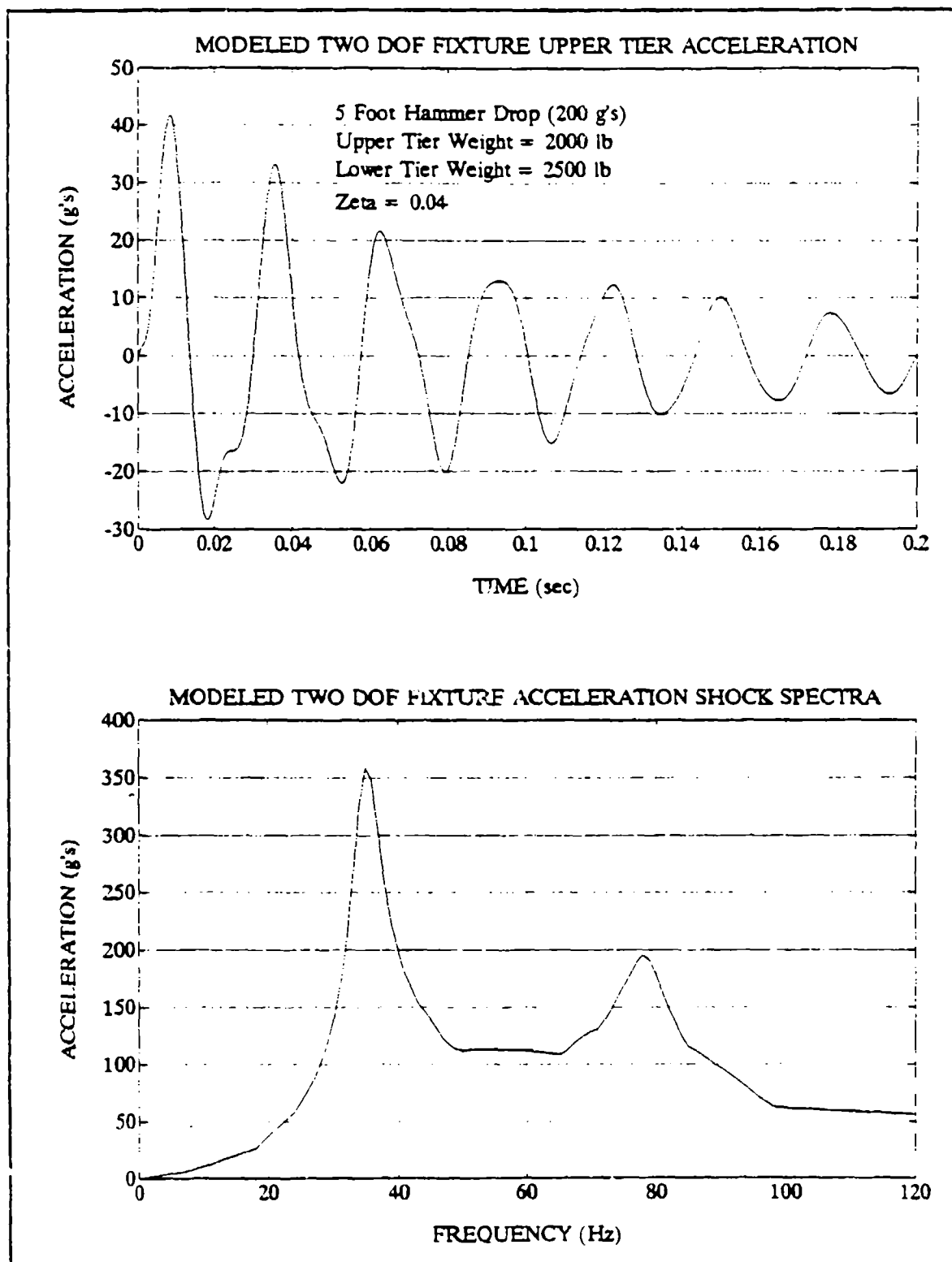


Figure 24. Predicted Acceleration Wave Form and Shock Spectrum for 2000 lb Upper Tier Weight.

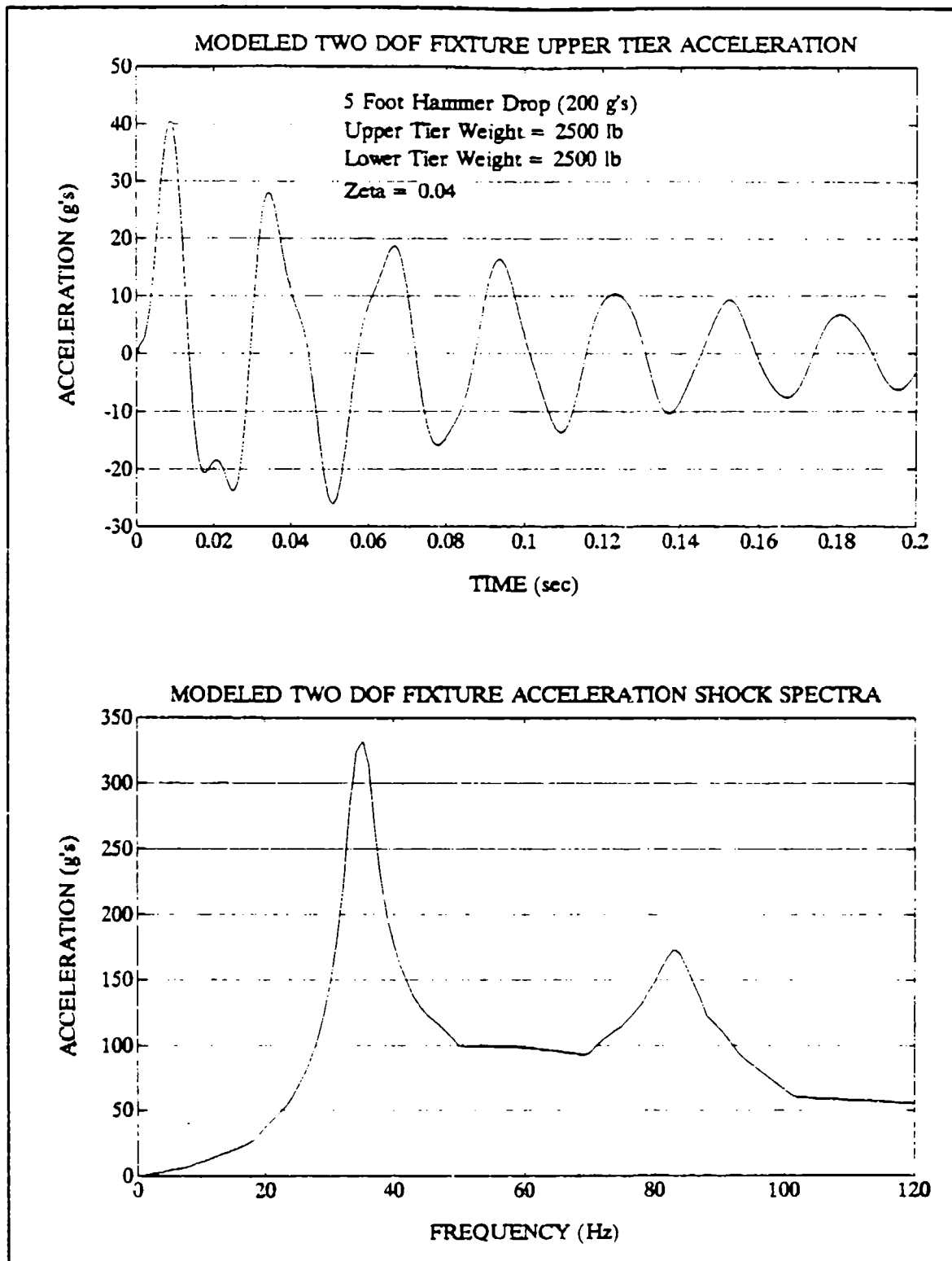


Figure 25. Predicted Acceleration Wave Form and Shock Spectrum for 2500 lb Upper Tier Weight.

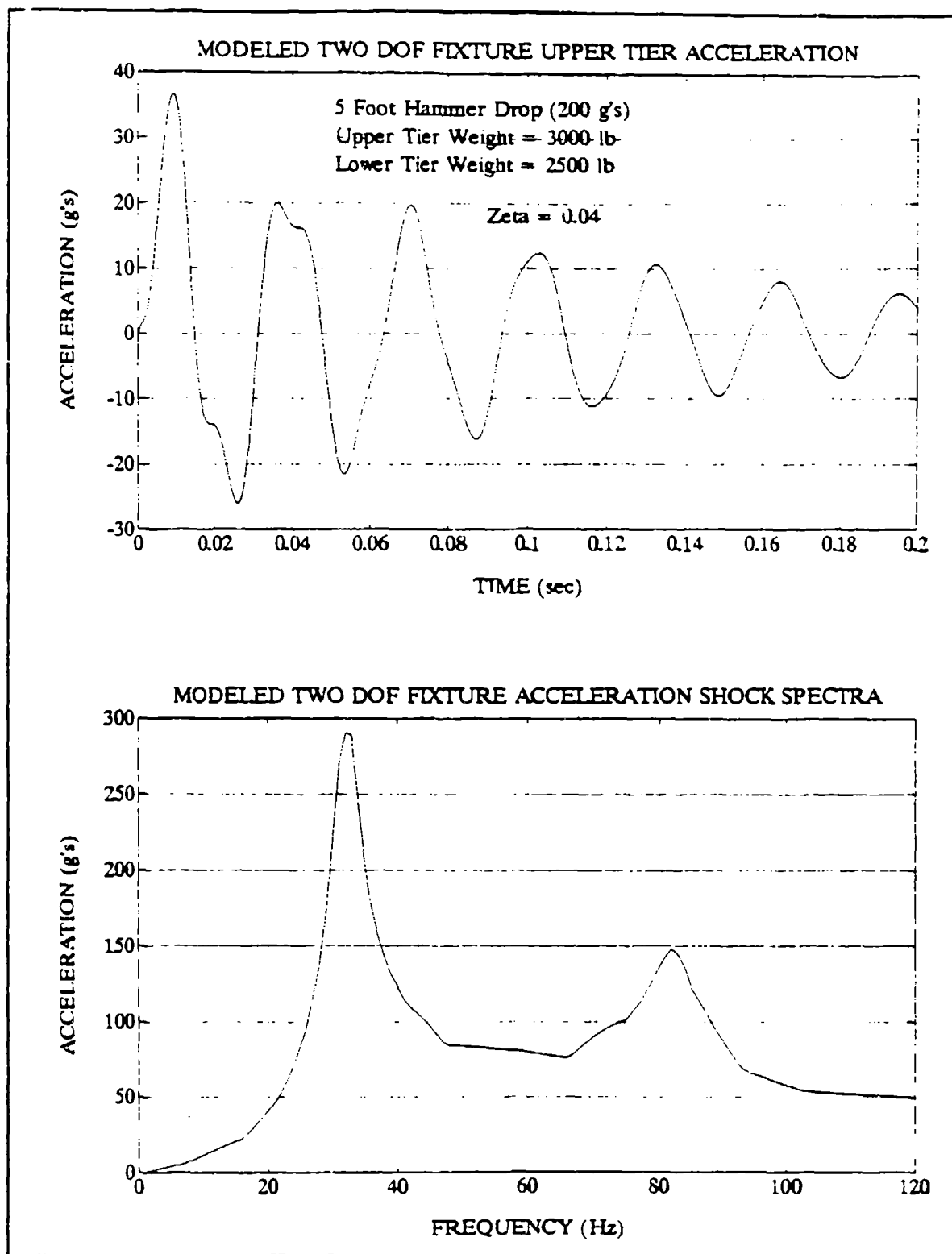


Figure 26. Predicted Acceleration Wave Form and Shock Spectrum for 3000 lb Upper Tier Weight.

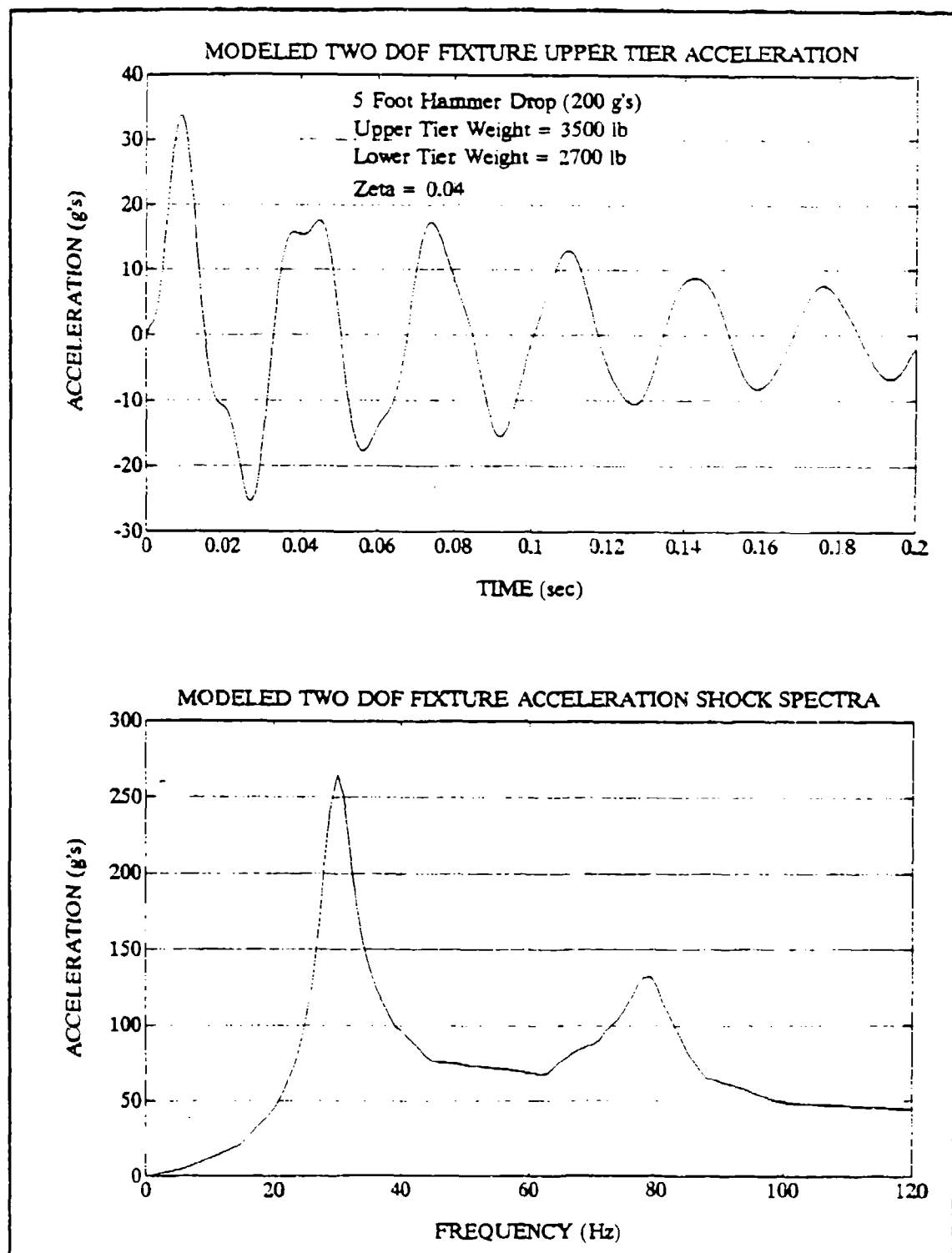


Figure 27. Predicted Acceleration Wave Form and Shock Spectrum for 3500 lb Upper Tier Weight.

IV. SEMI-DEFINITE THREE DEGREE-OF-FREEDOM MODEL

As mentioned in Chapter II, Corbell formulated a two-degree-of-freedom model to simulate the fixture response to the anvil table input. The fixture model was shown in Figure 8. The input into the model was the half-sine wave base acceleration impulse described in Figure 9. After the base impulse passed (1 msec), the anvil table was considered stationary and the fixture vibrated as a two DOF mass-spring-damper system attached to a fixed foundation. This model provided useful information required for determining the feasibility and design of the proposed fixture. In this chapter, a semi-definite three DOF model will be introduced which will take into account, the 4500 lb anvil table interaction with the fixture after the time of hammer impulse.

A. SEMI-DEFINITE THREE DOF MODEL DERIVATION

1. Model Description

As presented in Clements (1972), the anvil table is bolted to the machine foundation with 12, 2-inch-diameter bolts in a manner which allows the table three inches of vertical travel after hammer impact. This vertical distance can be decreased by raising the table with pneumatic jacks prior to performing the test. Approximately 50 msec after hammer impact, the table reaches the limit of vertical travel

and experiences a sharp "table reversal" in the form of a fairly simple negative acceleration pulse. Since the duration of the reversal is 2 to 4 times that of the hammer impact and energy has been expended in the system, the magnitude of the acceleration pulse is much less than that caused by the hammer. As the table falls back down on the foundation it experiences another impulse of longer duration and consequently lower acceleration. From the time of hammer impact to the time of table reversal, the anvil table is unconstrained and the entire structure behaves as the semi-definite three DOF system shown in Figure 28. Considering the initial impact as significantly more severe than the subsequent, it was possible to simplify the model and not include the table stop in the mathematical analysis for determining the characteristics of the fixture response.

Referring to Figure 28, the "upper tier" mass M_1 represents the mass of the equipment and mounting hardware and the effective spring mass for the upper tier. M_2 represents the intermediate mass and the effective spring mass for the lower tier. M_3 is the mass of the anvil table. The spring stiffness is designated by K_1 for the upper tier and K_2 for the lower. Again, as in the two DOF model, the structural damping is considered proportional to the mass and stiffness. The upper tier and lower tier damping is designated by C_1 and C_2 respectively.

THREE DOF MODEL

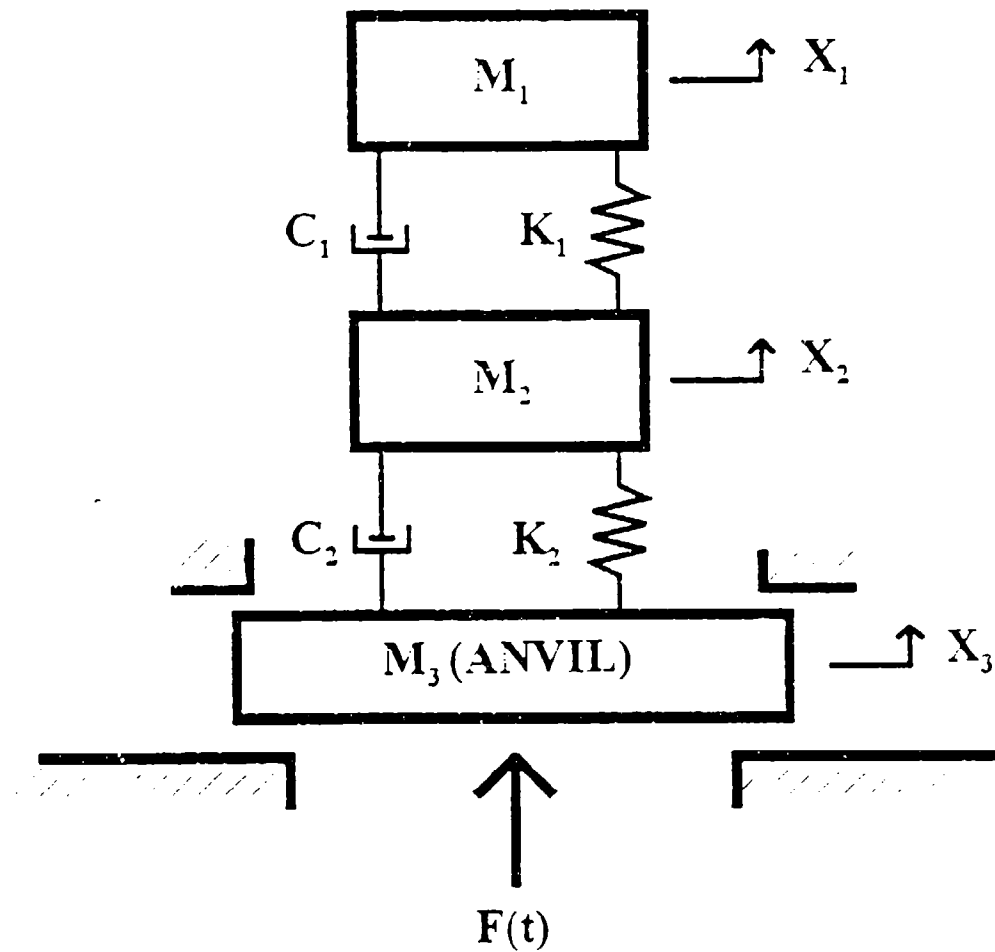


Figure 28. Three DOF Model for Fixture on MWSM.

Referring to Clements (1972), the peak anvil table acceleration produced by the half-sine impulse is very nearly a linear function of the hammer impact velocity and is essentially "independent" of the load when it is channel mounted. Using this information, the forcing function $F(t)$ can be modeled as simply the anvil acceleration times the anvil mass and is independent of the load attached.

$$F(t) = (Mass_{anvil}) \ddot{x}_3(t) \quad (18)$$

Since the test schedule for the MWSM listed in MIL-S-901D specifies the "hammer drop height" for a given table weight, it is convenient to write $F(t)$ as a function of the height of the hammer drop. Referring back to Figure 19, the 300 CPS filtered data shows a linear correlation between anvil acceleration and velocity with a slope of approximately 11.35 g·sec/ft. Using the relation, $V = \sqrt{2gh}$, the peak anvil table acceleration can be related to the hammer height by the following relation:

$$\ddot{x}_3(\text{peak}) = (11.35) (32.2) \sqrt{2gh} \quad (19)$$

where: h = Hammer Height Above Anvil Table in Feet

Knowing the peak acceleration of the half sine wave impulse, the forcing function $F(t)$ can then be modeled as:

$$\begin{aligned} F(t) &= 2933\sqrt{h}(M_3)\sin(\omega t) & 0 \leq t \leq 0.001 \text{ sec} \\ F(t) &= 0 & t > 0.001 \text{ sec} \end{aligned} \quad (20)$$

where: $\omega = 1000\pi$

The 1000 CPS filtered data shown in Figure 19 represents the table's 750 Hz longitudinal mode of damped vibration which last for only about 5 periods and does not significantly contribute to the motion of the fixture.

2. Mathematical Analysis of the Three DOF Model

The number of degrees of freedom of a system is defined as the number of independent coordinates necessary to describe the motion of a system completely. For the model shown in Figure 28, this will require three independent coordinates to define the motion of the three masses of the system. These coordinates, x_1 , x_2 and x_3 , define the absolute displacements of the masses, M_1 , M_2 and M_3 , respectively. The time derivatives of these coordinates yield the velocity and acceleration:

$$\begin{aligned} x_1, x_2, x_3 & \quad (\text{Displacement}) \\ \dot{x}_1, \dot{x}_2, \dot{x}_3 & \quad (\text{Velocity}) \\ \ddot{x}_1, \ddot{x}_2, \ddot{x}_3 & \quad (\text{Acceleration}) \end{aligned} \quad (21)$$

The resulting equations of motion in absolute coordinates can be written as:

$$m_1 \ddot{x}_1 + c_1 \dot{x}_1 + k_1 x_1 - c_1 \dot{x}_2 - k_1 x_2 = 0 \quad (22)$$

$$m_2 \ddot{x}_2 + (c_1 + c_2) \dot{x}_2 + (k_1 + k_2) x_2 - c_1 \dot{x}_1 - k_1 x_1 - c_2 \dot{x}_3 - k_2 x_3 = 0 \quad (23)$$

$$m_3 \ddot{x}_3 + c_2 \dot{x}_3 + k_2 x_3 - c_2 \dot{x}_2 - k_2 x_2 = F \quad (24)$$

The above equations of motion can be conveniently expressed in the following matrix form:

$$\begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix} \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_3 \end{bmatrix} + \begin{bmatrix} c_1 & -c_1 & 0 \\ -c_1 & (c_1 + c_2) & -c_2 \\ 0 & -c_2 & c_2 \end{bmatrix} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} + \begin{bmatrix} k_1 & -k_1 & 0 \\ -k_1 & (k_1 + k_2) & -k_2 \\ 0 & -k_2 & k_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ F \end{bmatrix} \quad (25)$$

Since the structural damping of the fixture is small, the system natural frequencies (free vibration) can be found by assuming that the damping is negligible and that the damped and undamped frequencies are equal. Neglecting damping and assuming free vibration, equation (25) can be rewritten as:

$$\begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix} \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_3 \end{bmatrix} + \begin{bmatrix} k_1 & -k_1 & 0 \\ -k_1 & (k_1 + k_2) & -k_2 \\ 0 & -k_2 & k_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad (26)$$

The system natural frequencies are then found by solving the following determinant for the system eigenvalues:

$$\det|[K] - \omega_n^2[M]| = 0 \quad (27)$$

$$\text{where } \omega_n = 2\pi f_n$$

The solution to equation (27) is shown below in equations (28) through (30) revealing that one of the three system natural frequencies is zero. This zero frequency corresponds to the rigid body mode of vibration.

$$\omega_{n1} = 0 \quad (\text{Rigid Body Mode}) \quad (28)$$

$$\omega_{n2} = \sqrt{\frac{1}{2} \left[B - \sqrt{B^2 - \frac{4K_1K_2}{M_1M_2M_3} (M_1 + M_2 + M_3)} \right]} \quad (29)$$

$$\omega_{n3} = \sqrt{\frac{1}{2} \left[B + \sqrt{B^2 - \frac{4K_1K_2}{M_1M_2M_3} (M_1 + M_2 + M_3)} \right]} \quad (30)$$

$$\text{where } B = \frac{K_1}{M_1} + \frac{K_2}{M_3} + \frac{K_1 + K_2}{M_2}$$

As described in Chapter II, if the desired system coupled natural frequencies f_{n1} , f_{n2} and f_{n3} are known, equations (29) and (30) can be iteratively solved to determine the required uncoupled tier natural frequencies f_1 and f_2 . A Mathcad® program is listed in Appendix C which, given the desired system natural frequencies will solve for the required tier frequencies.

Having found the system natural frequencies, the natural modes of vibration (eigenvectors) can be solved for using equation (31).

$$\begin{bmatrix} k_1 - \omega_{ni}^2 m_1 & -k_1 & 0 \\ -k_1 & (k_1 + k_2) - \omega_{ni}^2 m_2 & -k_2 \\ 0 & -k_2 & k_2 - \omega_{ni}^2 m_3 \end{bmatrix} \begin{bmatrix} \phi_{1,i} \\ \phi_{2,i} \\ \phi_{3,i} \end{bmatrix} = 0 \quad (31)$$

$i=1,2,3$

Equation (25) can be transformed into natural coordinates by introducing the transformation matrix $[\Phi]$ where the columns of $[\Phi]$ are the eigenvectors found from equation (31). Using the transforming matrix;

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \phi_{11} & \phi_{12} & \phi_{13} \\ \phi_{21} & \phi_{22} & \phi_{23} \\ \phi_{31} & \phi_{32} & \phi_{33} \end{bmatrix} \begin{bmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \end{bmatrix} \quad (32)$$

equation (25) can be decoupled by taking advantage of the orthogonality of the modal vectors. Applying the transformation matrix and premultiplying by $[\Phi]^T$, equation (25) can be rewritten as:

$$[\Phi]^T[M][\Phi]\{\ddot{\eta}\} + [\Phi]^T[C][\Phi]\{\dot{\eta}\} + [\Phi]^T[K][\Phi]\{\eta\} = [\Phi]^T[F] \quad (33)$$

Assuming that the modal damping is proportional to the mass and stiffness, equation (33) can be decoupled and written as three independent differential equations of motion as shown below:

$$\ddot{\eta}_1 + 2\zeta\omega_{n1}\dot{\eta}_1 + \omega_{n1}^2\eta_1 = F_1 \quad (34)$$

$$\ddot{\eta}_2 + 2\zeta\omega_{n2}\dot{\eta}_2 + \omega_{n2}^2\eta_2 = F_2 \quad (35)$$

$$\ddot{\eta}_3 + 2\zeta\omega_{n3}\dot{\eta}_3 + \omega_{n3}^2\eta_3 = F_3 \quad (36)$$

Equations (34) through (36) were solved for displacement, velocity and acceleration using the MATLAB™ program provided in Appendix D. As in Chapter II, the critical damping ratio was assumed to be 0.04. The solutions in natural coordinates was transformed back to real coordinates by reapplying equation (32).

B. THREE DOF FIXTURE MODEL RESPONSE

The upper tier acceleration response to the half sine wave impulse was modeled for the same weight and stiffness configurations as presented for the two DOF model in Figures 23 through 27. The modeled acceleration time response and shock spectra for five different upper tier weights are shown in Figures 29 through 33. As can be seen from the figures, the system natural frequencies for the three DOF model are approximately 15 to 20 Hz higher than for the two DOF model and peak accelerations are less. Figures 34 through 36 show the time response and shock spectra comparisons of the two DOF and three DOF models.

Since the anvil table is not absolutely unconstrained due to friction on the bolts and contact with the stops, it is expected that the actual system natural frequencies will fall somewhere between those of the two DOF and three DOF models. In the next chapter, the results of the 1/4 scale model testing will be presented which will help better predict the response of the actual fixture.

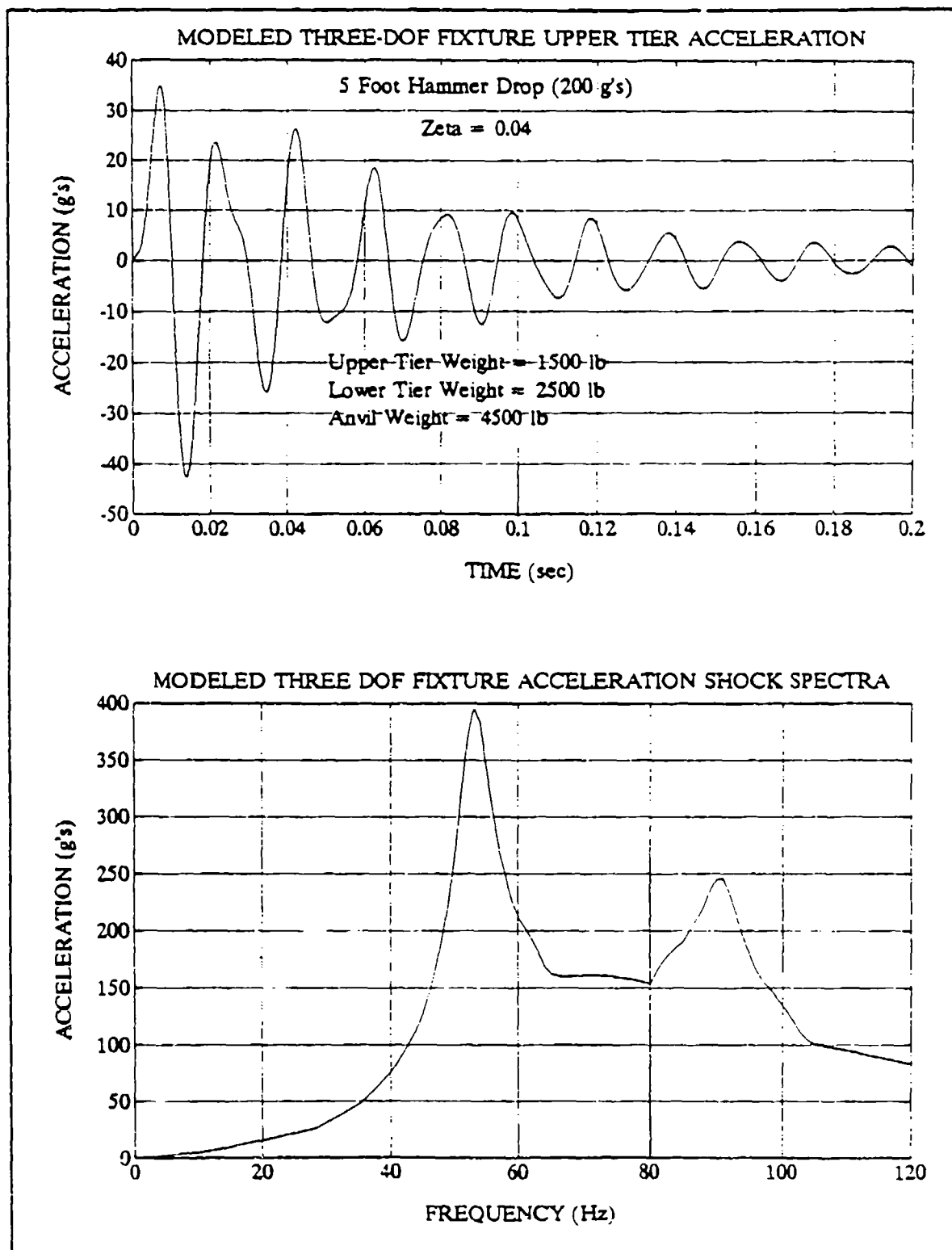


Figure 29. Three DOF Model Acceleration Wave Form and Shock Spectrum for 1500 Lb Upper Tier Weight.

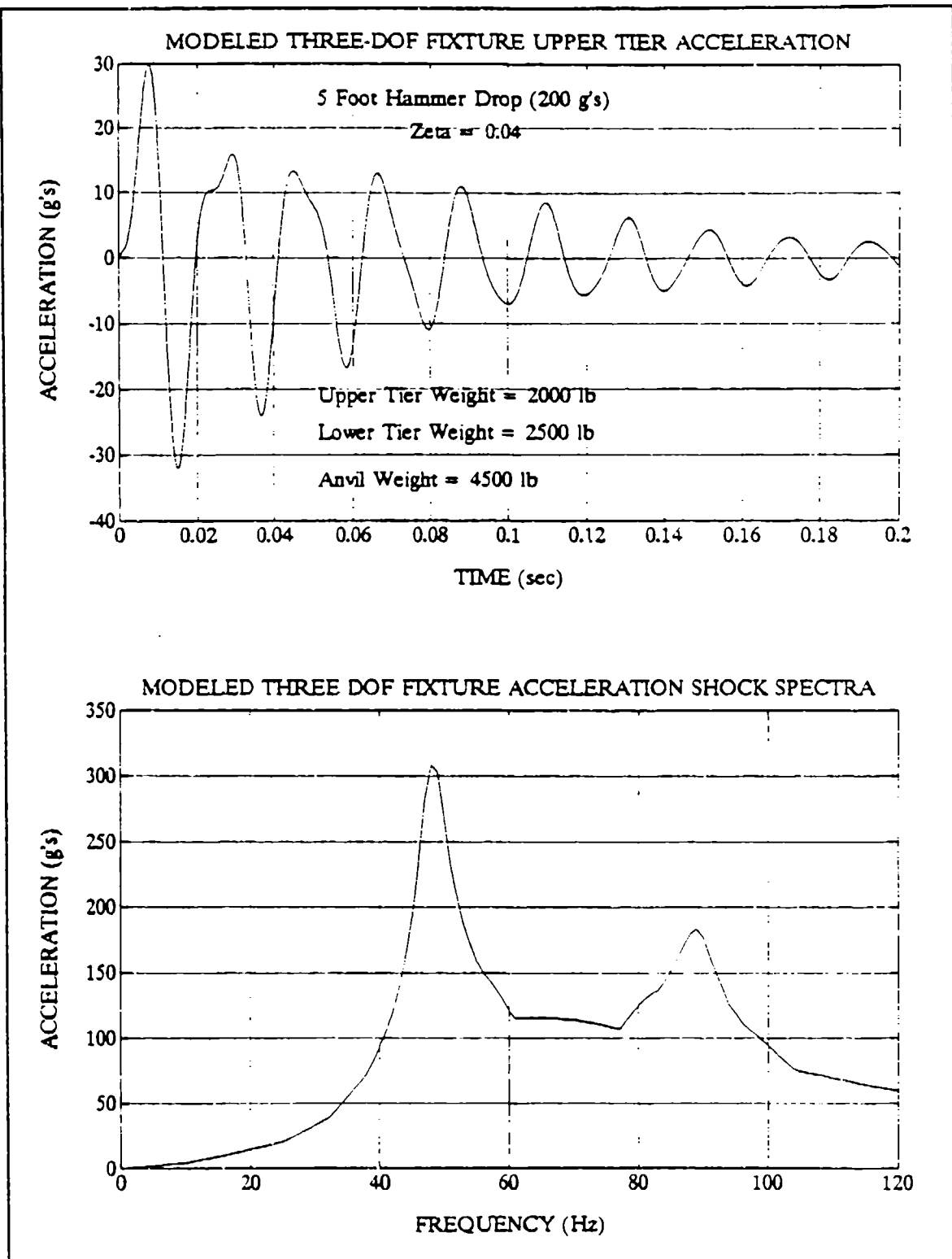


Figure 30. Three DOF Model Acceleration Wave Form and Shock Spectrum for 2000 Lb Upper Tier Weight.

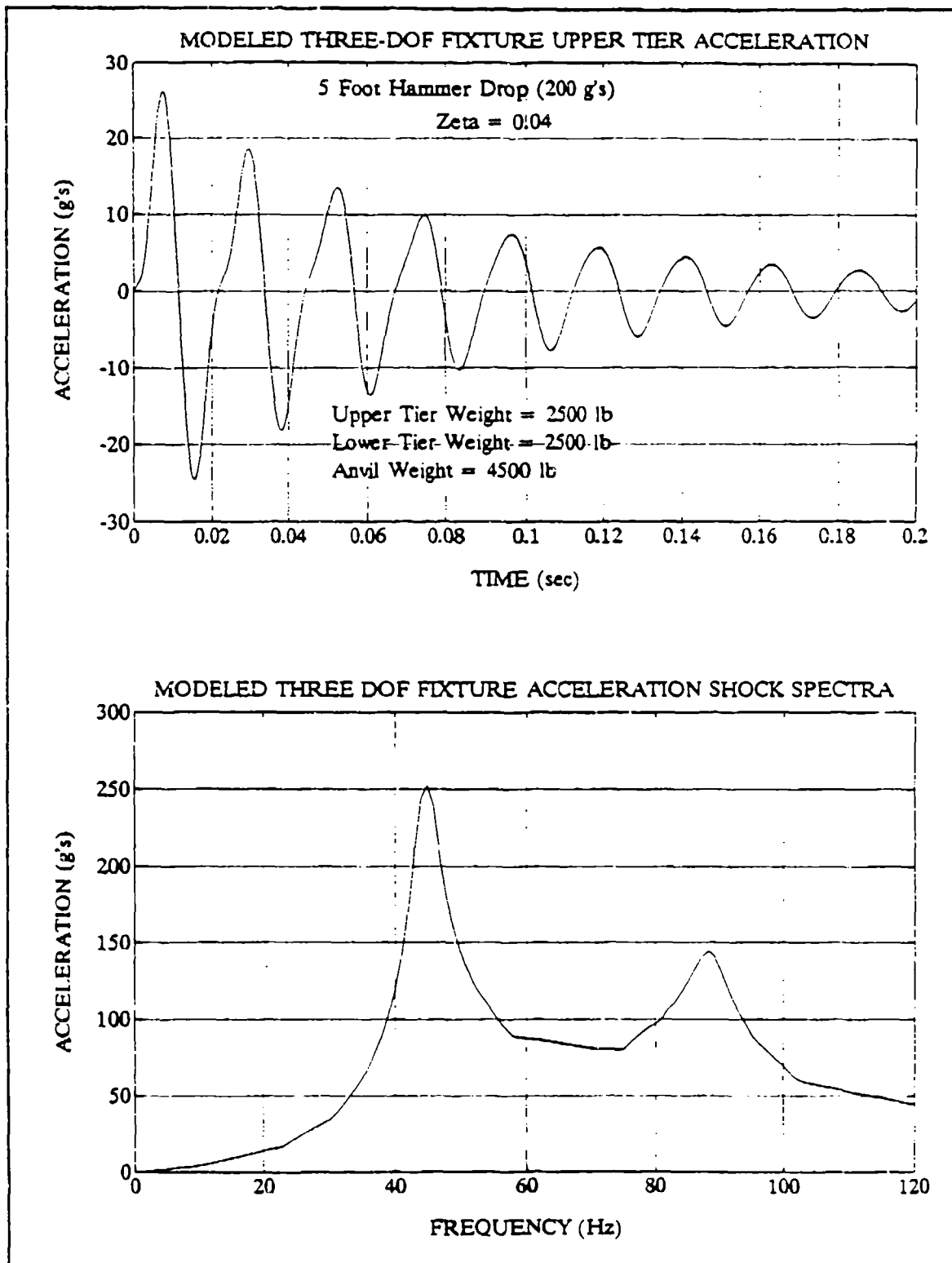
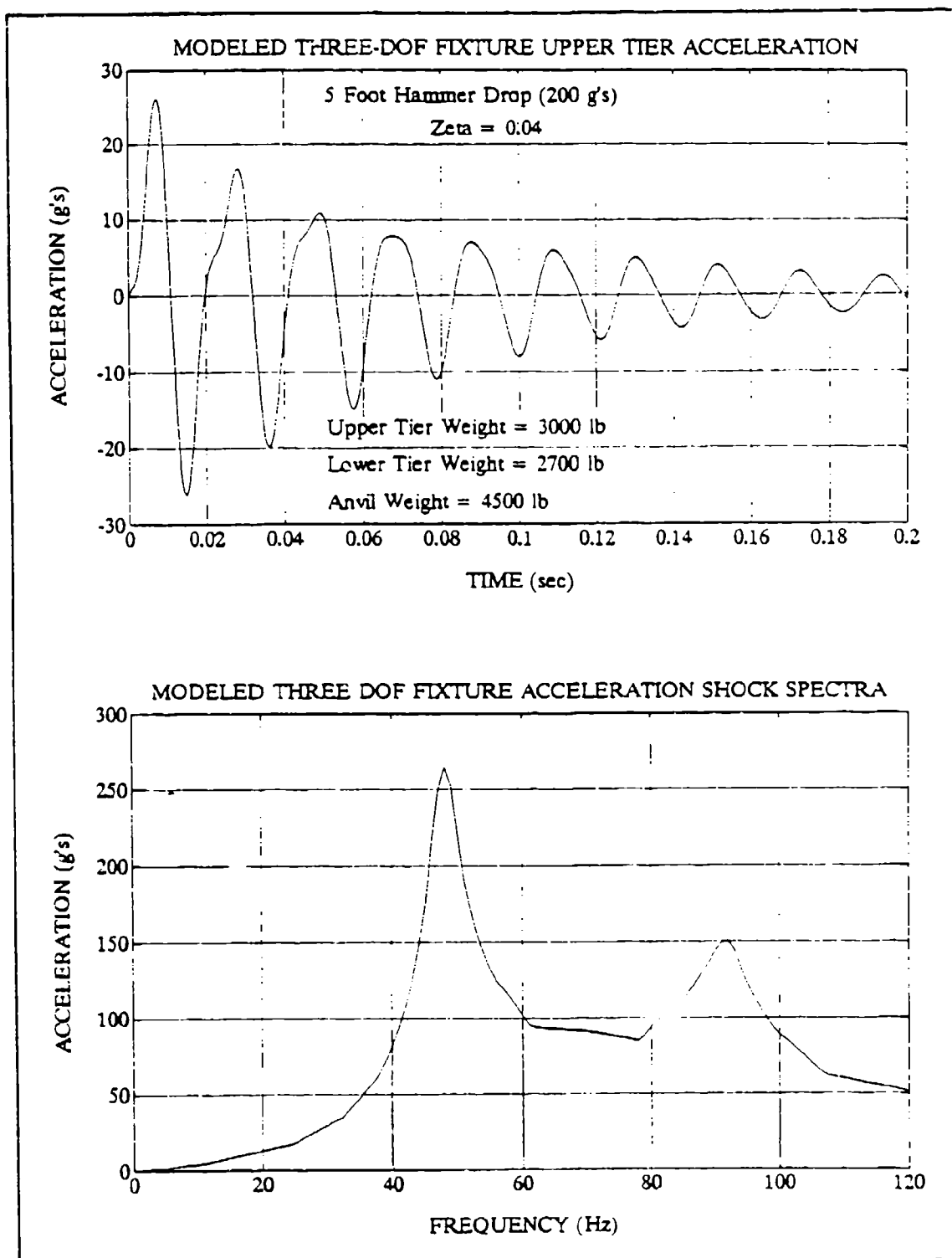


Figure 31. Three DOF Model Acceleration Wave Form and Shock Spectrum for 2500 Lb Upper Tier Weight.



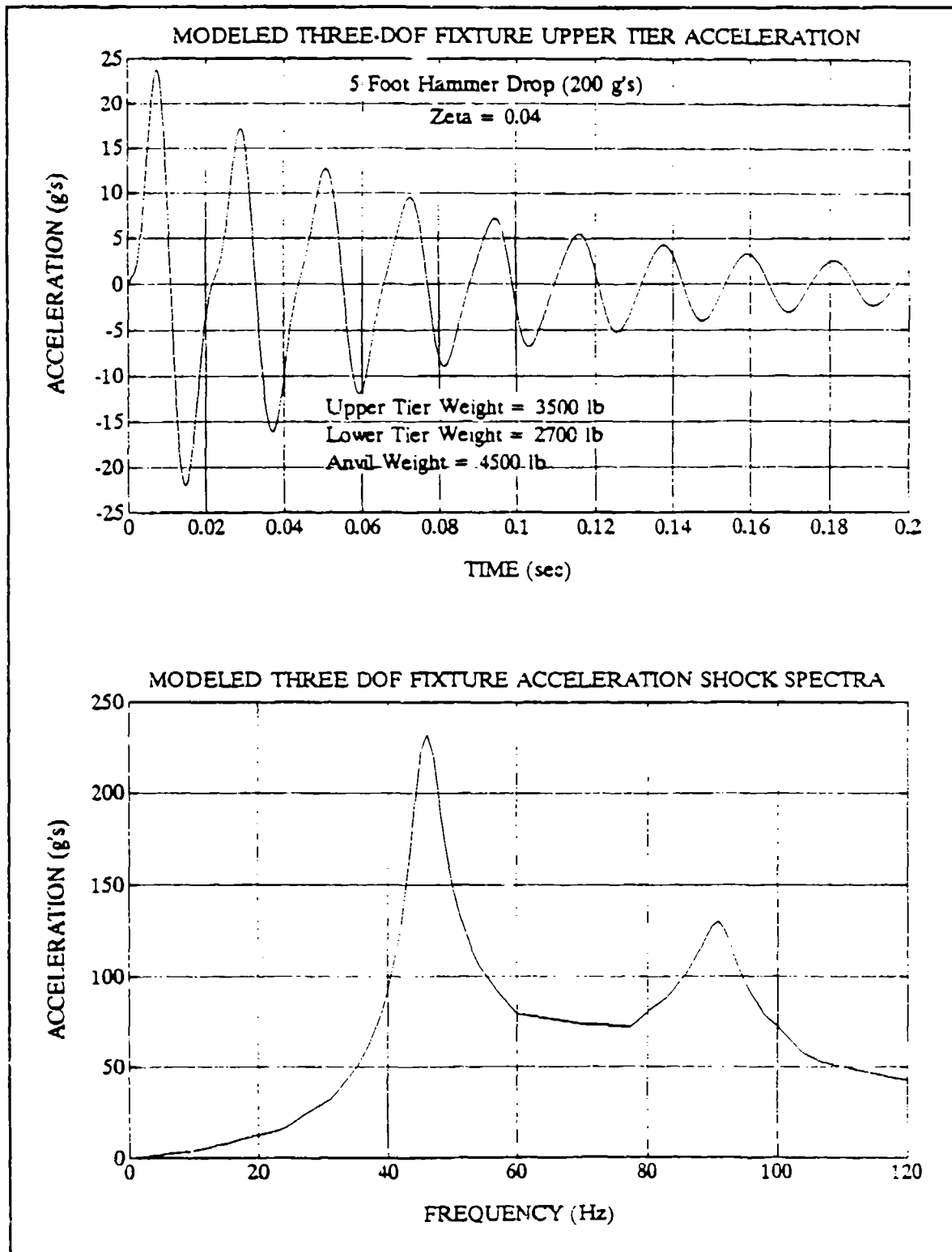


Figure 33. Three DOF Model Acceleration Wave Form and Shock Spectrum for 3500 Lb Upper Tier Weight.

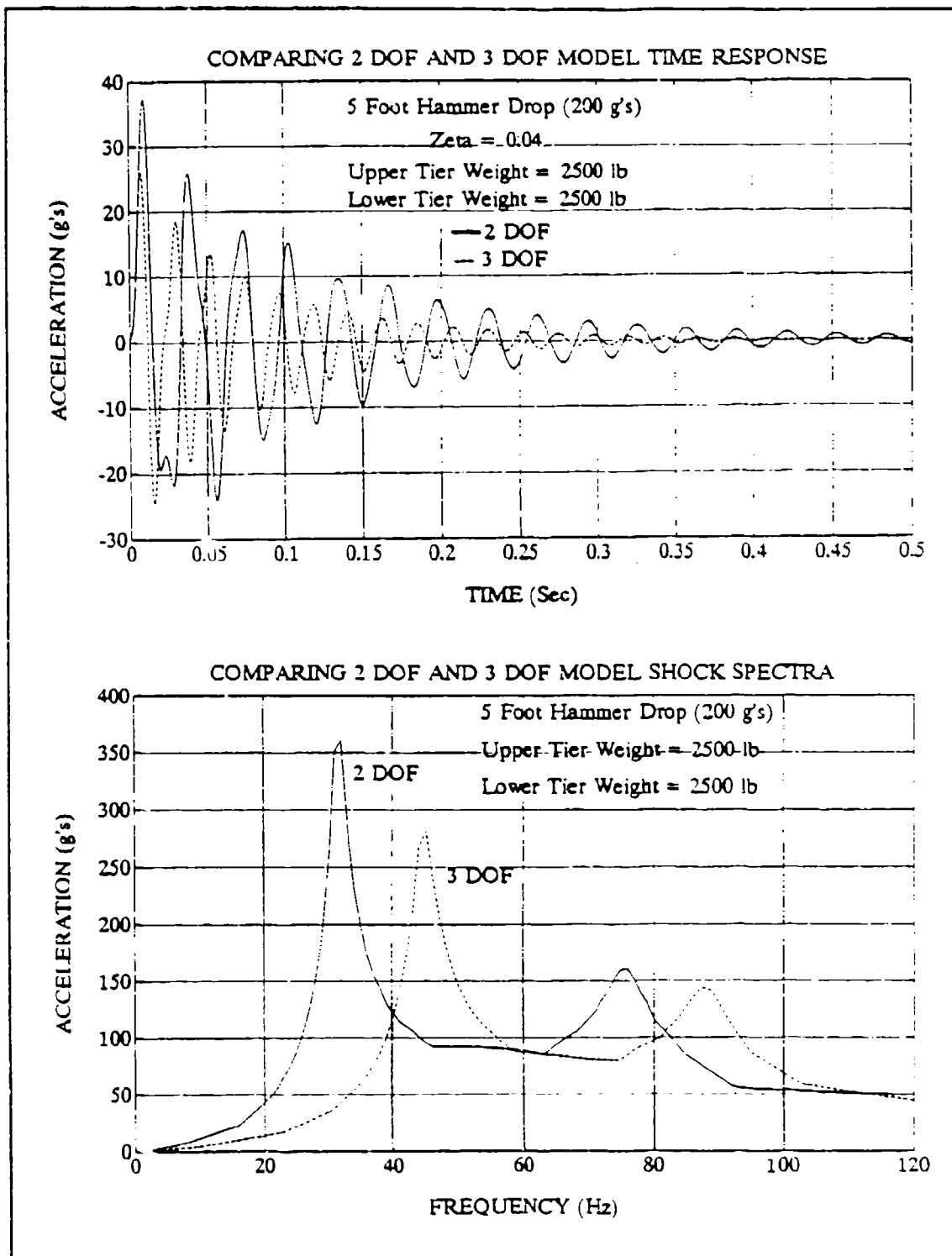


Figure 34. 2 DOF and 3 DOF Model Time Response and Shock Spectra Comparisons for 2500 Lb Upper Tier Weight.

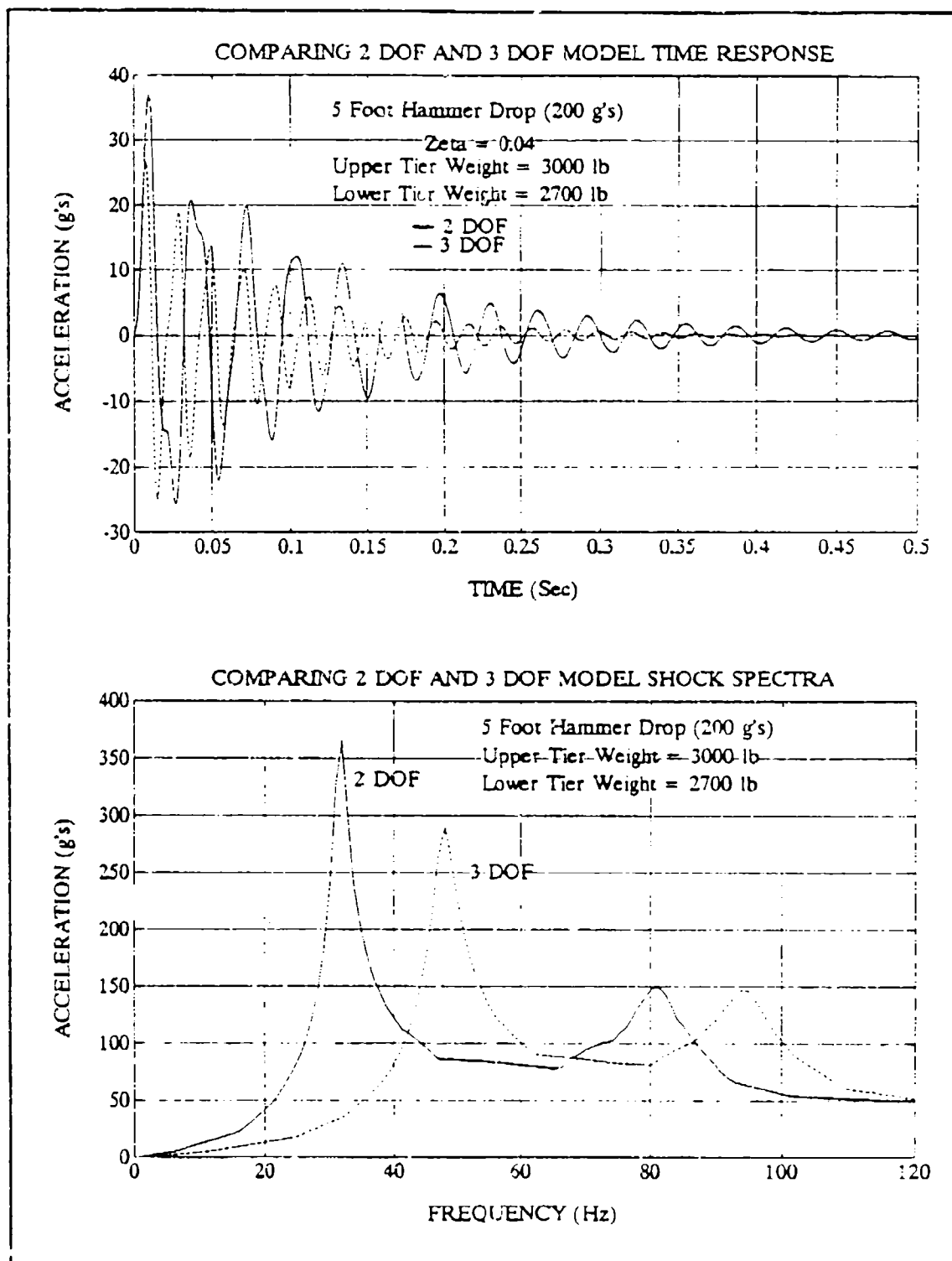


Figure 35. 2 DOF and 3 DOF Model Time Response and Shock Spectra Comparison for 3000 Lb Upper Tier Weight.

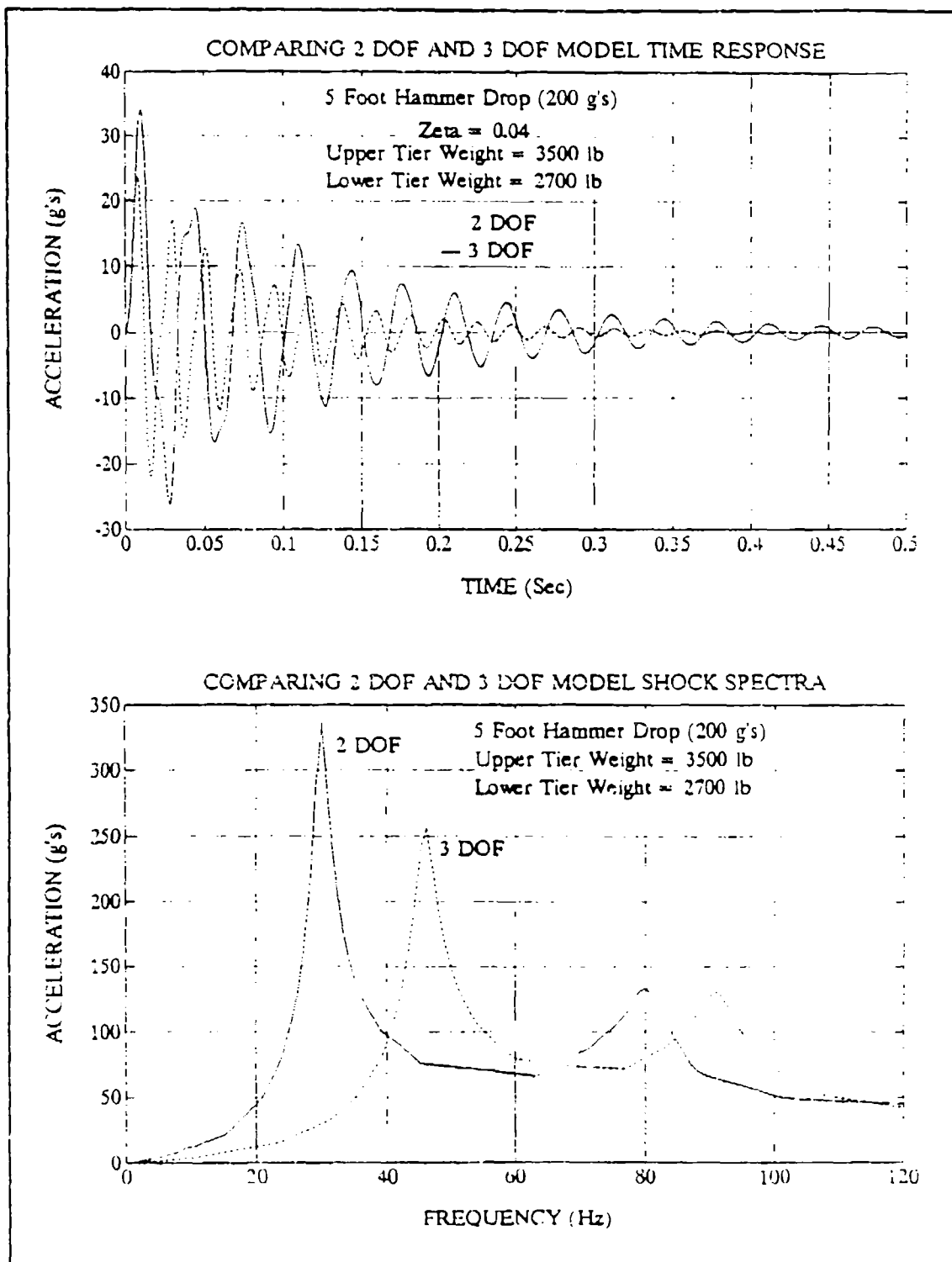


Figure 36. 2 DOF and 3 DOF Model Time Response and Shock Spectra Comparison for 3500 Lb Upper Tier Weight.

V. 1/4 SCALE MODEL TESTING AND RESULTS

To verify the mathematical simulation, a 1/4 scale model of the fixture was fabricated for shock testing. The principal intent of the testing was to confirm the analytical methods employed were appropriate for the application and that the expected frequencies characteristics could be obtained. The following sections describe the test procedure used and the results obtained.

A. EXPERIMENTAL SET UP

To supply the impulse excitation to the model, a drop table was constructed which allowed dropping the fixture model onto a load cell from various heights. The range of the load cell was 0-5000 lbs with a sensitivity of 0.106 mv/lb. The upper tier accelerometer had a range of 0-500 g's with a sensitivity of 9.98 mv/g. Using a Hewlet Packard 3562A Dynamic Signal Analyzer, the transfer function between the input excitation and upper tier acceleration was obtained. The experimental set up is shown in figure 37 and a schematic of the instrumentation is shown in figure 38.

B. 1/4 SCALE MODEL

The fixture model was built as closely as possible to a 1/4 scale version of the proposed fixture. Exceptions



Figure 37. 1/4 Scale Model and Drop Table.

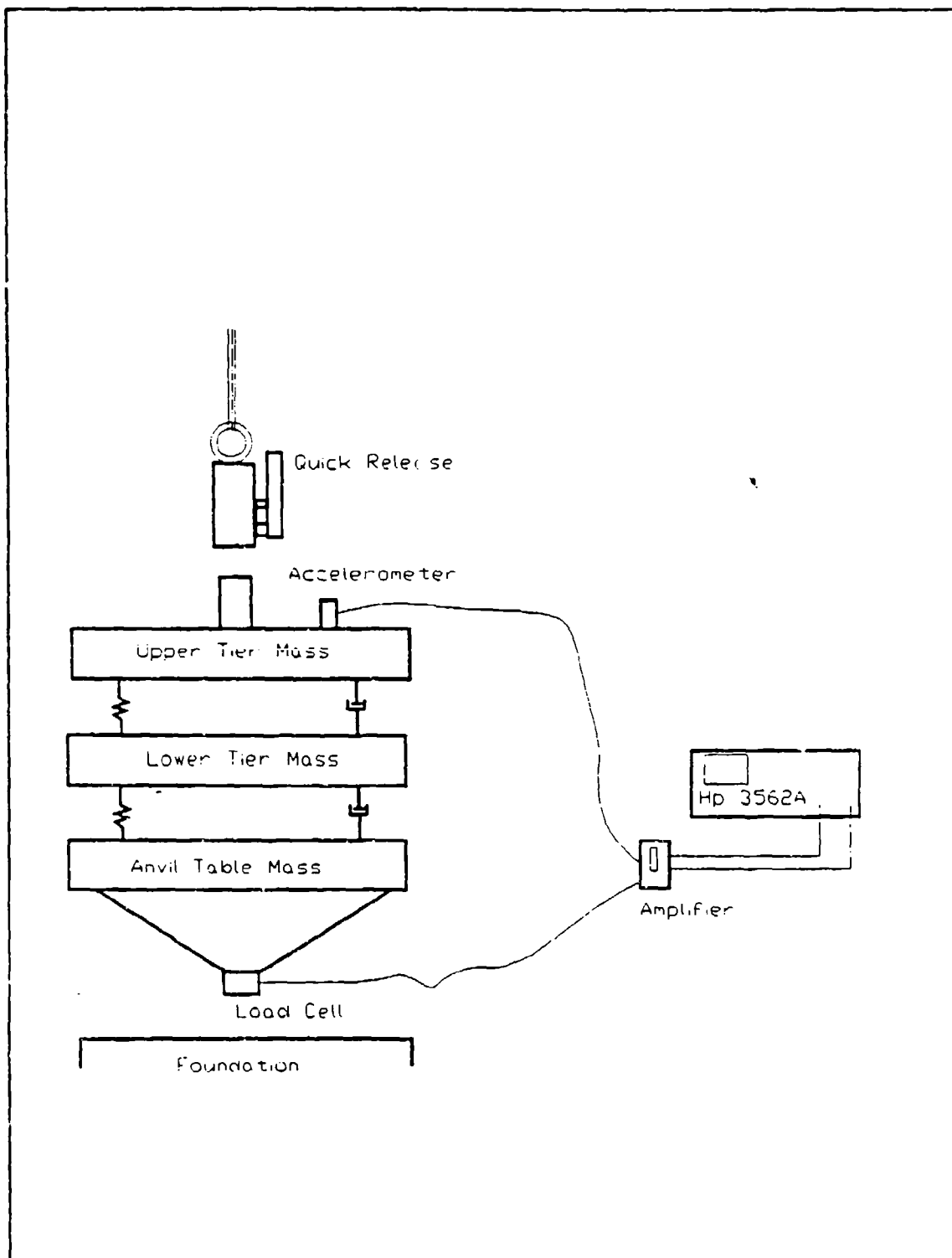


Figure 38. Instrumentation Setup for Drop Testing 1/4 Scale Model Test Fixture.

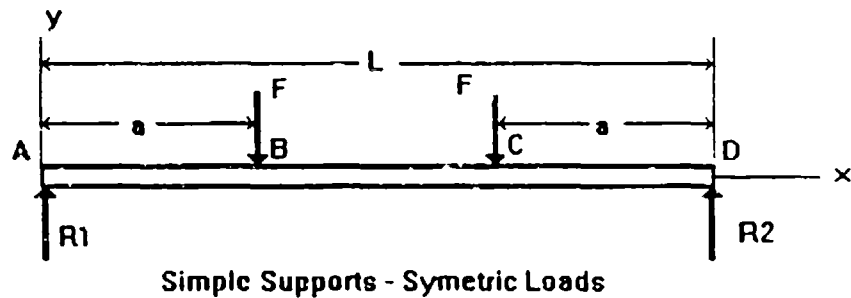
included using aluminum instead of steel for the spring I-beams and an additional 1/4 scale reduction in the weight of the tiers. The upper tier weight as tested was 43.5 lbs, the lower tier weight was 55.2 lbs and the anvil table was 109.1 lbs. The per-beam-stiffness of the aluminum I-beams was 2.09×10^4 lbs/in.

To help verify the boundary conditions used in the mathematical model, the spring beams were mounted using the same clamping design proposed for the full scale prototype. The following example provides the dimensions and calculations for determining the expected system natural frequencies for the three DOF model.

Example: Determine System Natural Frequencies For 1/4 Scale Model Fixture:

A. SPRING STIFFNESS:

The spring stiffness is based on the following beam model:



1) Dimensions:	<u>Upper Tier</u>	<u>Lower Tier</u>
Length:	$L_1 = 12.5$ in	$L_2 = 12.5$ in
Load Application Point:	$a_1 = 3.25$ in	$a_2 = 3.25$ in
Area Moment of Inertia:	$I_1 = 0.045$ in ⁴	$I_2 = 0.045$ in ⁴
Young's Modulus:	$E = 10 \cdot 10^6$ psi	$E = 10 \cdot 10^6$ psi

2) **Stiffness:**

From the beam bending equation: $y_{AB} = \frac{F \cdot x}{6 \cdot EI} \cdot (x^2 + 3 \cdot a^2 - 3 \cdot L)$

Let $x=a$ and $F=W/2$: $y_{AB} = \frac{W \cdot a^2}{12 \cdot EI} \cdot (4 \cdot a - 3 \cdot L)$

Solving for the stiffness: $K = \frac{W}{y_{AB}} = \frac{12 \cdot EI}{a^2 \cdot (3 \cdot L - 4 \cdot a)} \quad \frac{\text{lb}}{\text{in}}$

For:

Upper Tier

Lower Tier

(Convert inches to feet) $KB_1 = \frac{12 \cdot EI_1}{a_1^2 \cdot (3 \cdot L_1 - 4 \cdot a_1)} \cdot 12$ $KB_2 = \frac{12 \cdot EI_2}{a_2^2 \cdot (3 \cdot L_2 - 4 \cdot a_2)} \cdot 12$

(per beam) $KB_1 = 2.504 \cdot 10^5 \frac{\text{lb}}{\text{ft}}$

$KB_2 = 2.504 \cdot 10^5 \frac{\text{lb}}{\text{ft}}$

Upper Tier

Lower Tier

Input Number of Beams Per Tier: $Beams_1 = 2$

$Beams_2 = 2$

$K_1 = KB_1 \cdot Beams_1$

$K_2 = KB_2 \cdot Beams_2$

Tier Stiffness: $K_1 = 5.008 \cdot 10^5 \frac{\text{lb}}{\text{ft}}$

$K_2 = 5.008 \cdot 10^5 \frac{\text{lb}}{\text{ft}}$

B. SYSTEM MASSES: (Includes Effective Spring Mass)

Upper Tier Weight: (W_1)

Intermediate Weight: (W_2)

Anvil Table Weight: (W_3)

$W_1 = 43.5$

$W_2 = 55.2$

$W_3 = 109.1$

$M_1 = \frac{W_1}{32.2}$

$M_2 = \frac{W_2}{32.2}$

$M_3 = \frac{W_3}{32.2}$

$M_1 = 1.351 \frac{\text{lb} \cdot \text{ft}}{\text{sec}^2}$

$M_2 = 1.714 \frac{\text{lb} \cdot \text{ft}}{\text{sec}^2}$

$M_3 = 3.388 \frac{\text{lb} \cdot \text{ft}}{\text{sec}^2}$

C. TIER NATURAL FREQUENCIES:

Upper Tier

$$F_1 = \frac{1}{2\pi} \sqrt{\frac{K_1}{M_1}}$$

$$F_1 = 96.904 \text{ Hz}$$

Lower Tier

$$F_2 = \frac{1}{2\pi} \sqrt{\frac{K_2}{M_2}}$$

$$F_2 = 86.023 \text{ Hz}$$

D. System Natural Frequencies:

Defining B as:

$$B = \frac{K_1}{M_1} + \frac{K_2}{M_3} + \frac{K_1 - K_2}{M_2}$$

$$f_{n1} = \frac{1}{2\pi} \sqrt{\frac{1}{2} \left[B - \sqrt{B^2 - \frac{4K_1K_2}{M_1M_2M_3} (M_1 - M_2 + M_3)} \right]}$$

First Frequency: $f_{n1} = 77.762 \text{ Hz}$

$$f_{n2} = \frac{1}{2\pi} \sqrt{\frac{1}{2} \left[B + \sqrt{B^2 - \frac{4K_1K_2}{M_1M_2M_3} (M_1 - M_2 + M_3)} \right]}$$

Second Frequency: $f_{n2} = 147.944 \text{ Hz}$

C. EXPERIMENTAL RESULTS

The first experiment performed was determining the static stiffness of an assembled tier containing three spring beam elements. Using the simply supported beam model, the expected stiffness of the tier was calculated. The tier was then placed on a load machine and loaded up to 1000 lbs while measuring

the static deflection. Figure 39 shows the analytical stiffness versus the experimental. From the plot, the actual stiffness was measured to be 43,500 lb/in which is about 5 percent higher than the analytical stiffness. This result tends to justify the simply supported boundary conditions used in the numerical model.

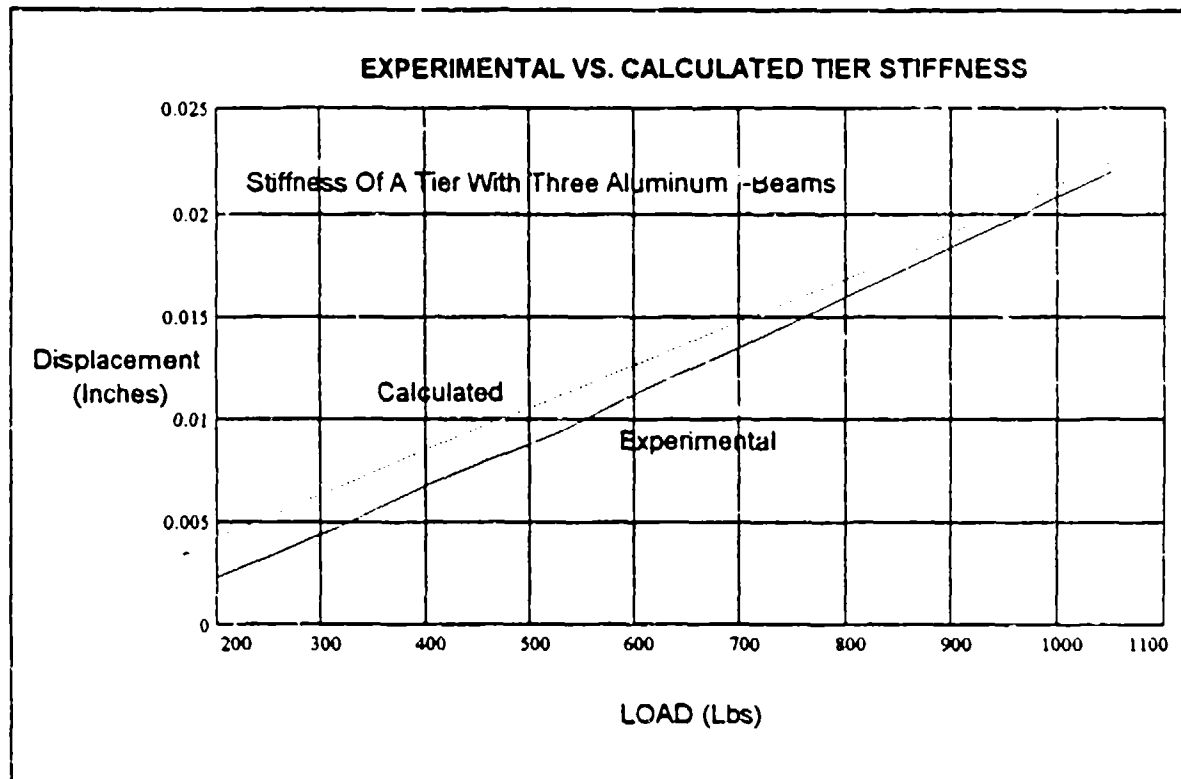


Figure 39. Experimental Tier Stiffness versus Analytical.

The model was then installed on the drop table to determine its response to a six inch table drop. Using the load cell as the input and upper tier acceleration as the output, the transfer function for a ten drop average was computed using the HP 3562A. The acceleration data was

filtered using an exponential window with a period $T=400$ msec. The data collection was triggered off the load cell input using a -5 msec trigger delay. The transfer function with phase shift is shown in Figure 40. The experimental transfer function reveals distinct peaks at 80 Hz and 160 Hz with associated 180 degree phase shifts. These results agree very closely with the mathematical predictions of $f_{n1}=77.8$ Hz and $f_{n2}=148$ Hz.

Figure 41 shows the unfiltered load cell and acceleration output sampled at 10,000 Hz. The load cell data shows a maximum impulse of approximately 35,000 lbs with a duration of about 10 msec. The corresponding upper tier acceleration is 170 g's. Using a logarithmic decrement as an approximation, the critical damping ratio was determined to be two percent.

The experimental results, although not extensive, support the mathematical model very well. They demonstrate that using the proposed fixture will transform the high frequency, high impact energy of the hammer-anvil impact into discrete equipment excitation frequencies, tunable to shipboard conditions. With full scale fixture testing beginning in July 1993, promising results are expected.

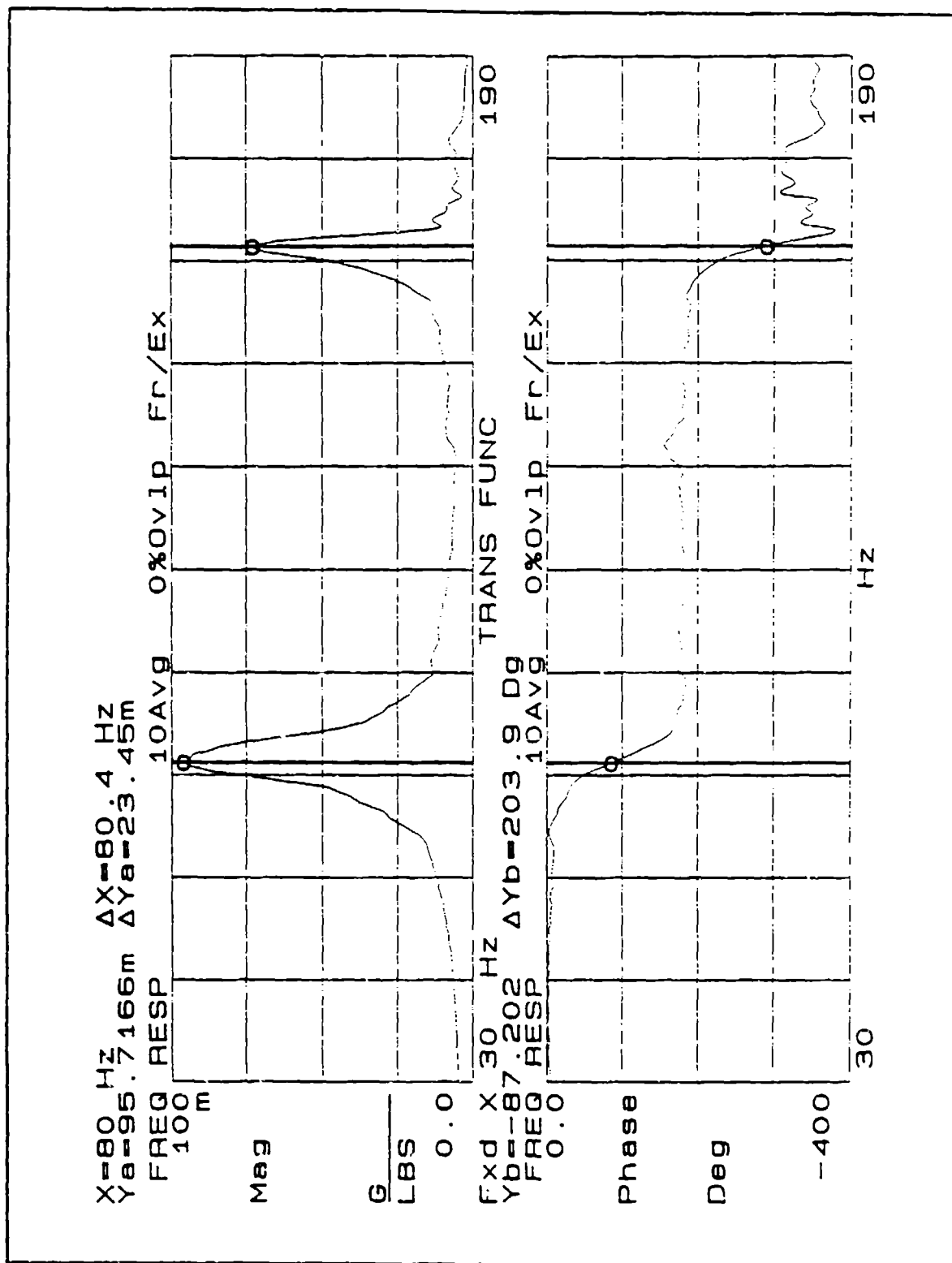


Figure 40. 1/4 Scale Model Fixture Transfer Function and Phase Shift.

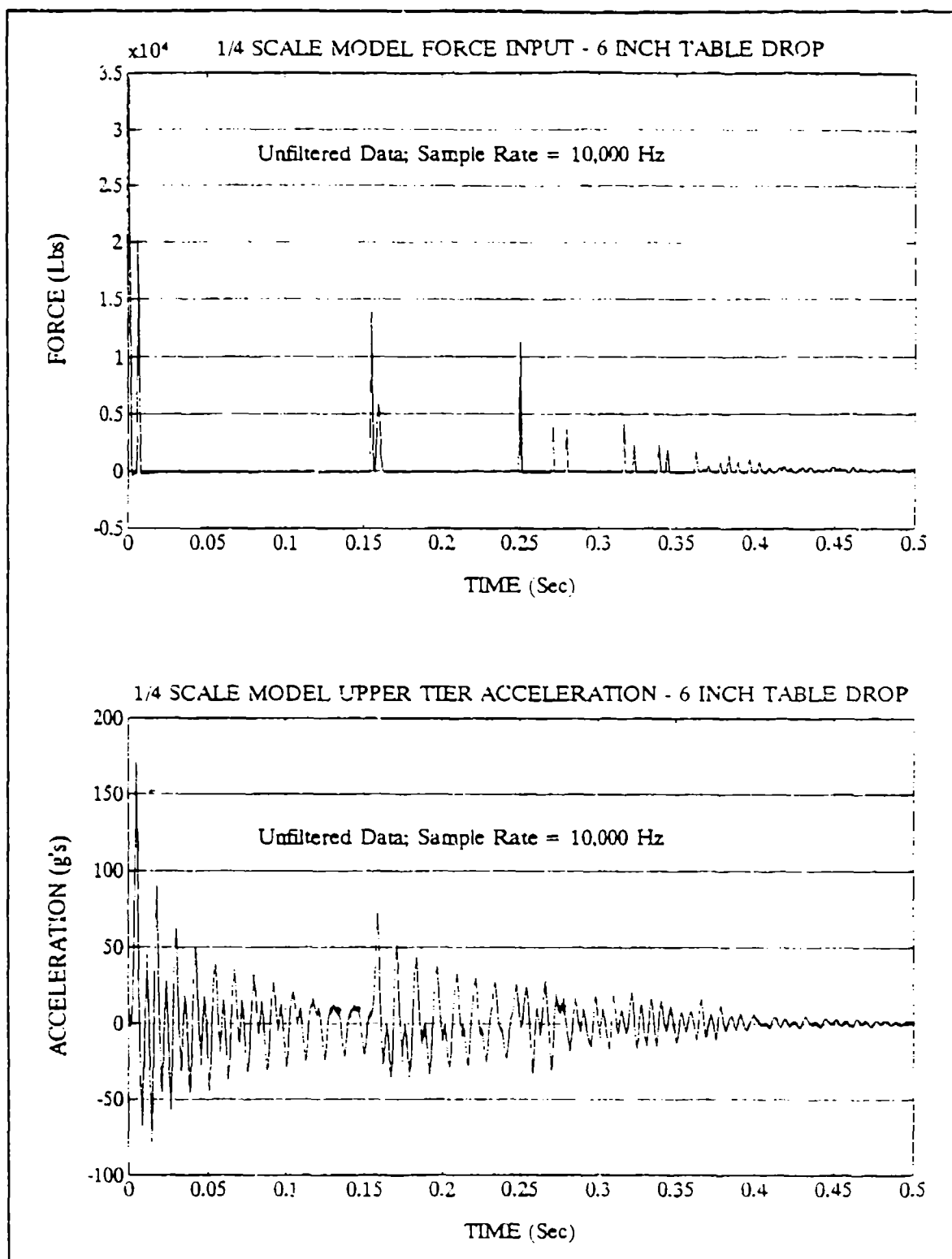


Figure 41. 1/4 Scale Model Unfiltered Load Cell and Accelerometer Output. Sample Frequency 10,000 Hz.

VI. CONCLUSIONS AND RECOMMENDATIONS

The mathematical model and experimental results provided in this study demonstrate great promise in the concept of using a tuned two DOF equipment mounting fixture on the MWSM. Using frequencies provided by NSWC, the mathematical model provided the required stiffness and mass parameters necessary to design the fixture. The design was then scaled to one quarter and an experimental model constructed for impact testing. The results of the model testing agreed extremely well with the results predicted from the mathematical modeling. The following is a list of specific comments concerning the tuned fixture:

- The fixture designed in this study is not expected to be an all inclusive tuned fixture capable of testing all mediumweight equipment. Its specific design is to provide excitation at 30 Hz and 80 Hz for equipment up to 3500 lbs. As more information becomes available on localized ship structure excitation, a series of tuned fixtures can be constructed (economically), each designed for a specific location on the ship.
- Since the weight capacity of the MWSM is limited to 7400 lbs, equipment weight using a two DOF fixture will be limited to approximately 3500 lbs. The ratio between the upper and lower tier mass is limited requiring them to be nearly equivalent in magnitude. From the DTRC/UERD

predictions, this should not be a significant problem since heavier equipment will not generally require a two DOF fixture to simulate shipboard shock.

- The I-beam spring design will work very well for frequencies above 25 Hz, but would not be suitable for lower excitation frequencies. Since the spring beams are limited to approximately sixty inches in length on the MWSM, the vertical tier displacements required for lower frequencies would result in excessive bending stress in the I-beams. For this reason, it is recommended that the coil spring design be reinvestigated for lower frequencies.
- On completion of the prototype fixture in June 1993, a series of calibration tests will be performed to determine the actual response of the fixture. Based on these test, the mathematical model can be updated with structural damping characteristics as well as adjusting the boundary conditions if necessary. This new model will then be available for the design of future fixtures.

The use of the two DOF equipment mounting fixture for shock qualifying equipment on the MWSM is in line with the Navy's goal of shock hardening ships. Incorporating this concept into the current shock qualification procedures will provide increased reliability of essential equipment subjected to shock from underwater explosions.

APPENDIX A. TWO DOF TUNED FIXTURE PROGRAM

```

% Program TWODOF.M
% Lt. David M. Cox
% Naval Postgraduate School

% This MATLAB program plots the relative displacement,
% relative velocity and absolute acceleration of a TDOF mass,
% spring, damper system subjected to base excitation (half-sine
% pulse). Additionally, the shock spectra of an undamped SDOF
% system is calculated using the TDOF upper tier acceleration
% as the base excitation to the SDOF system.

clear

%User Defined Variables

G=203           ; % Acceleration Magnitude in g's
Wt1=3500        ; % Weight of Upper Tier in Lbs
Wt2=2700        ; % Weight of Lower Tier in Lbs
kb=2.6e6        ; % Stiffness per Beam
k1=3*kb;        ; % Upper Tier Stiffness in Lbs/ft
k2=4*kb;        ; % Lower Tier Stiffness in Lbs/ft
zeta=.04        ; % Critical Damping coefficient

%Formulate the Base Acceleration
dt=0.0001       ; % time step for sampling
w=1000*pi       ; % base excitation frequency
N=5000;         ; % Number of steps
T=N*dt          ; % Total time record
t=[0:dt:(N-1)*dt] ; % Time vector from t=0 to t=T
for i=1:N       ; % Loop to produce half-sine wave pulse
if t(i)<=0.001
    anvil(i) = G*32.2*sin(w*t(i));
else
    anvil(i) = 0;
end
end

%Calculate the mass and mass ratio
m1=Wt1/32.2 ;
m2=Wt2/32.2 ;
m=[m1,0;0,m2];
alpha=m1/m2;

```

```

%Calculate Circular Tier Natural Frequencies
w1=sqrt(k1/m1);
w2=sqrt(k2/m2);
%System Natural Frequencies
var=sqrt((w1^2 + alpha*w1^2 + w2^2)^2 - (2*w1*w2)^2)
wn1=(1/sqrt(2))*sqrt(w1^2 + alpha*w1^2 + w2^2 - var)
wn2=(1/sqrt(2))*sqrt(w1^2 + alpha*w1^2 + w2^2 + var)
%Uncouple the equations
u11=1;
u21=(k1 - wn1^2 * m1)/k1;
u12=1;
u22=(k1 - wn2^2 * m1)/k1;
U=[u11,u12;u21,u22];
%Decouple the system
M=U'*m*U
%Stiffness Matrix
k=[k1,-k1;-k1,k1+k2]; % coupled stiffness
K=U'*k*U
%Force Coefficients
FC=(U'*[-m1;-m2])
%Solve the uncoupled Equations Of Motion
% {nddot1}+[2*zeta*wn1]*{ndot1}+[wn1^2]*{n1}=FC11*anvil/M1
% {nddot2}+[2*zeta*wn2]*{ndot1}+[wn2^2]*{n2}=FC21*anvil/M2

%State Space Matrix for Equation #1
A1=[0,1;-(wn1^2),-2*zeta*wn1]
B1=[0;FC(1:1)/M(1:1)];
C1=[1,0;0,1;-(wn1^2),-2*zeta*wn1];
D1=[0;0;FC(1:1)/M(1:1)];
[y1]=lsim(A1,B1,C1,D1,anvil,t);
%State Space Matrix for Equation #2
A2=[0,1;-(wn2^2),-2*zeta*wn2]
B2=[0;FC(2:2)/M(4:4)];
C2=[1,0;0,1;-(wn2^2),-2*zeta*wn2];
D2=[0;0;FC(2:2)/M(4:4)];
[y2]=lsim(A2,B2,C2,D2,anvil,t);

%Couple The Equations
Y1DIS=(u11*y1(:,1) + u12*y2(:,1))*12;
Y1VEL=u11*y1(:,2) + u12*y2(:,2);
Y1ACC=u11*y1(:,3) + u12*y2(:,3);
%Absolute acceleration in g's
X1ACC=(Y1ACC+anvil')/32.2;
%Convert to in.
Y2DIS=(u21*y1(:,1) + u22*y2(:,1))*12;
Y2VEL=u21*y1(:,2) + u22*y2(:,2);
Y2ACC=u21*y1(:,3) + u22*y2(:,3);
X2ACC=(Y2ACC+anvil')/32.2;

```

```

plot(t,X1ACC),grid
title('MODELED TWO DOF FIXTURE UPPER TIER ACCELERATION')
xlabel('TIME (sec)')
ylabel('ACCELERATION (g''s)')
gtext('5 Foot Hammer Drop (200 g''s)')
gtext('Upper Tier Weight = 3500 lb')
gtext('Lower Tier Weight = 2700 lb')
gtext('Zeta = 0.04')
meta fig28.met

plot(t,Y1DIS,t,Y2DIS),grid
xlabel('TIME(Seconds)')
ylabel('RELATIVE DISPLACEMENT (Inches)')
title('MODELED TIER DISPLACEMENT RELATIVE TO THE ANVIL TABLE')
gtext('Hammer Height = 5 feet (200 g''s)')
gtext('1 - Upper Tier Weight = 3500 lb')
gtext('2 - Lower Tier Weight = 2700 lb')
gtext('Zeta = 0.04')
gtext('1'),gtext('2')
meta fig20.met
pause

%Calculate the shock spectra
NF=120; % Number of Frequencies
DF=1; % Frequency Increment
SF=1; % Start Frequency
Freq=[SF:DF:NF]; % Frequency Vector
wn=2*pi*Freq;
F=[0;-1]; % State Space Matrices
GG=[1,0];
H=[-1];
for i=1:NF % Step Thru Natural Frequencies
E=[0,1;-(wn(i)^2),0];
GG=[wn(i)^2,0];
[yspec]=lsim(E,F,GG,H,X1ACC,t);
xspec = yspec + X1ACC;
maxspec(i)=max(abs(xspec));
end

plot(Freq,maxspec)
title('MODELED TWO DOF FIXTURE ACCELERATION SHOCK SPECTRA')
xlabel('FREQUENCY (Hz)')
ylabel('ACCELERATION (g''s)')
grid
gtext('fn1 = 38 Hz')
gtext('fn2 = 78 Hz')
meta fig28.met

```

APPENDIX B. TUNED FIXTURE DESIGN DRAWINGS

DESIGN DRAWINGS

ITEMIZED PART LIST

Piece Number	Item	Size (inches)	No. Required
1	Shipbuilding Channel	7 x 22.7#	2
2	I-Beam	W5 x 18.5#	4
3	I-Beam	W8 x 67#	3
4	Block	1" x 2 1/4" x 10"	28
5	Stiffner	See Figure 2 Sheet 2	6
6	I-Beam	W4 x 13#	7
7	Stiffner	See Figure 2 Sheet 5	12
8	Block	1" x 2" x 5 1/8"	28
9	Clamp	See Figure 4 Sheet 1	14
10	Clamp	See Figure 5 Sheet 1	28
11	Washer	1 1/8" ID x 2" OD	84
12	Hexagon Head Bolt	1"-8 x 4 1/4" Long	84

Notes:

- 1) Top flange of piece number 1 shall be burned off or cut off to a width of 1 3/4".
- 2) Piece numbers 9 and 10 will be shaped to fit inner surface of piece numbers 1 and 6 respectively.
- 3) Hexagon head bolts may be replaced with equivalent allen head bolts to ease installation of piece 10 to piece 2. Depending on weld thickness and assembly tolerance, use of a socket to tighten hexagon head bolts may be difficult.

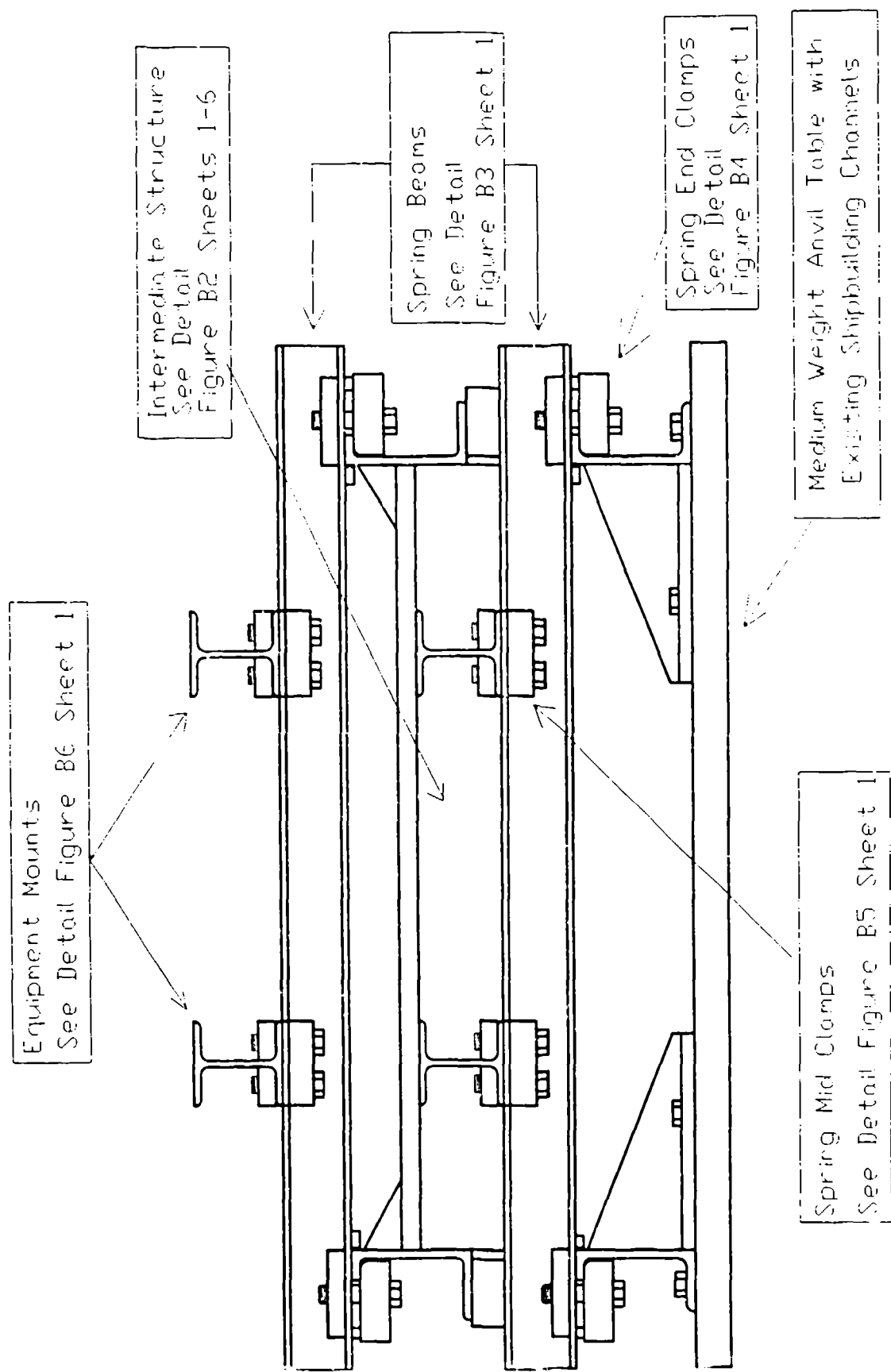
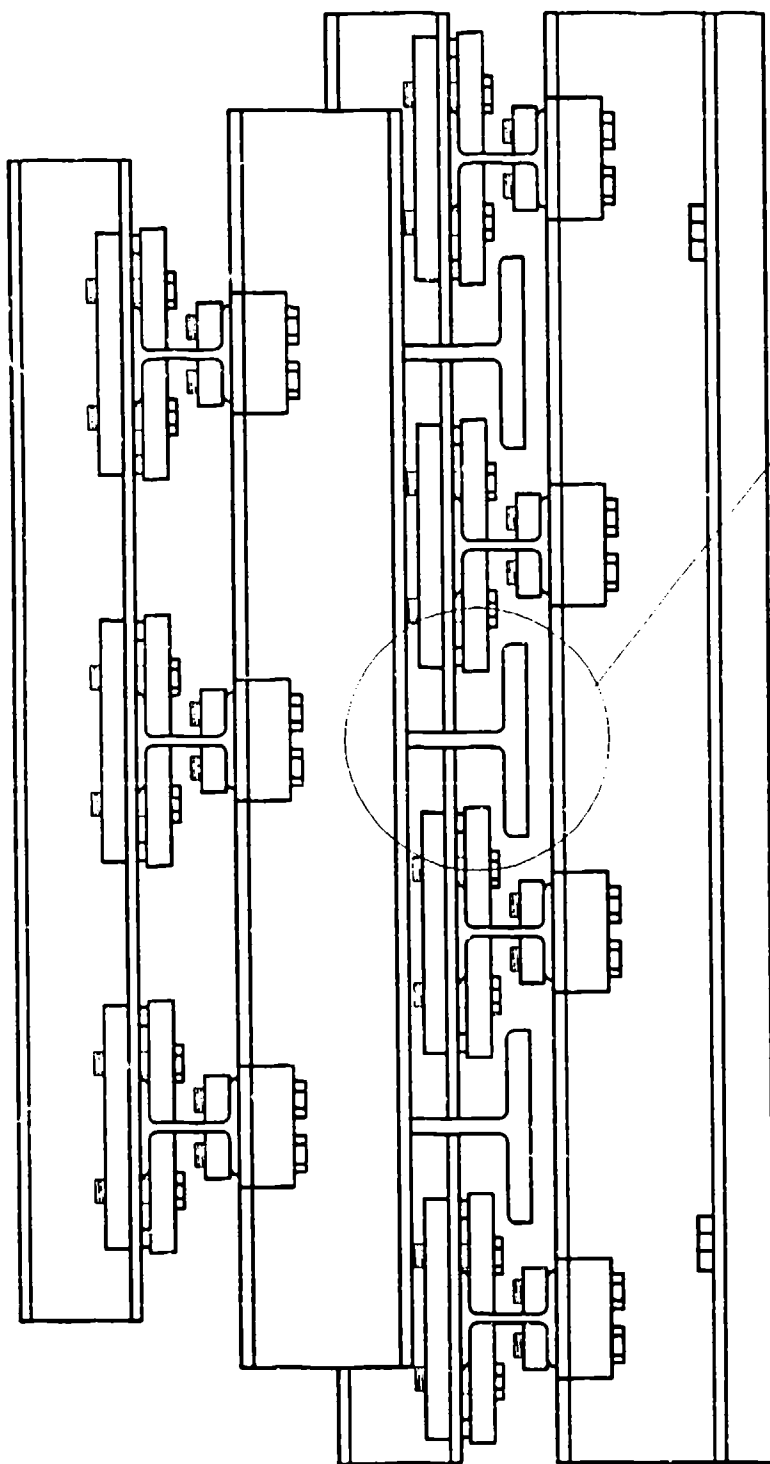


Figure B1 Sheet 1: Two DOF Fixture Front View



Note: Stiffeners omitted for clarity
See Detail Figure B2 Sheet 5

Figure B1 Sheet 2: Two DDF Fixture Side View

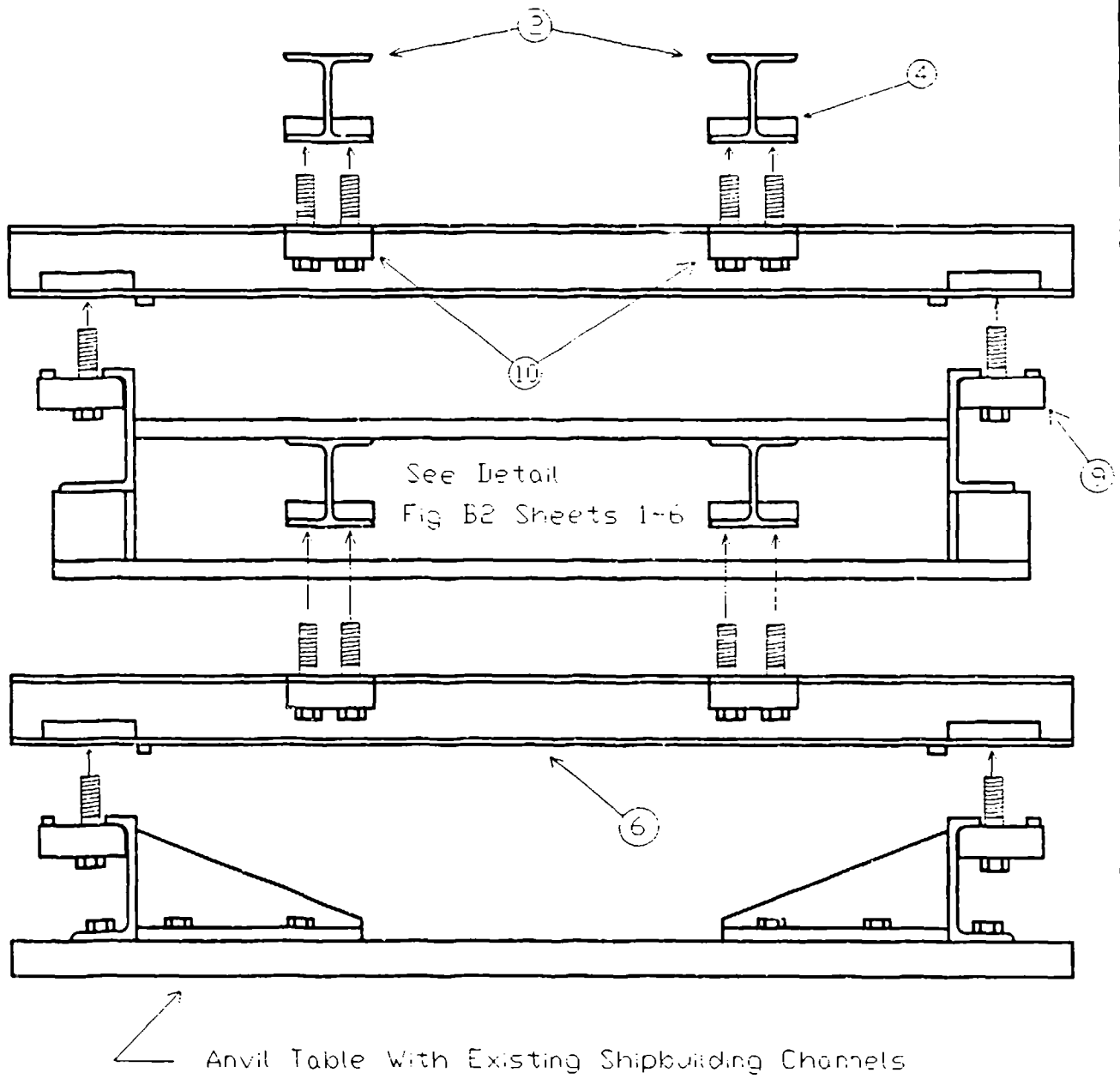
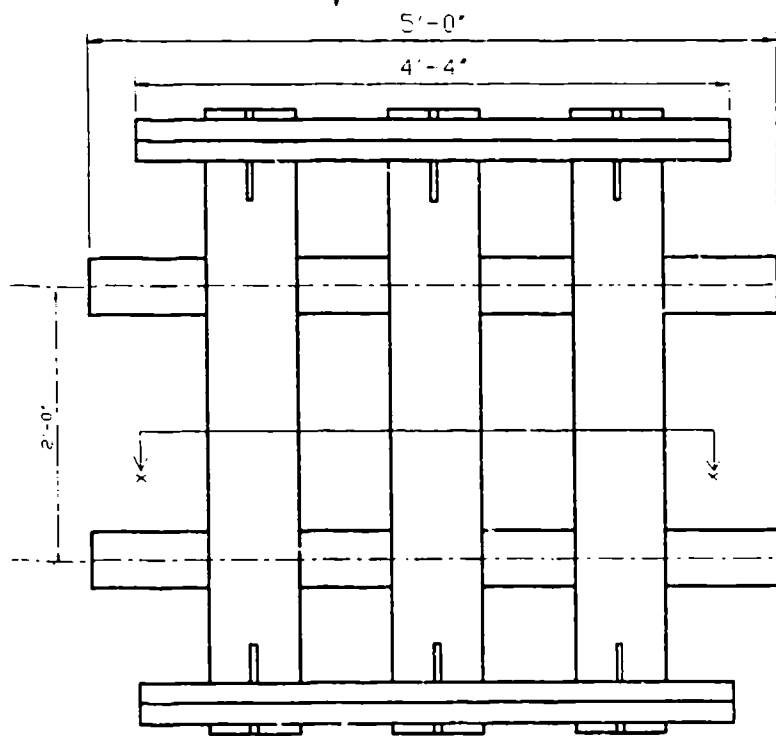
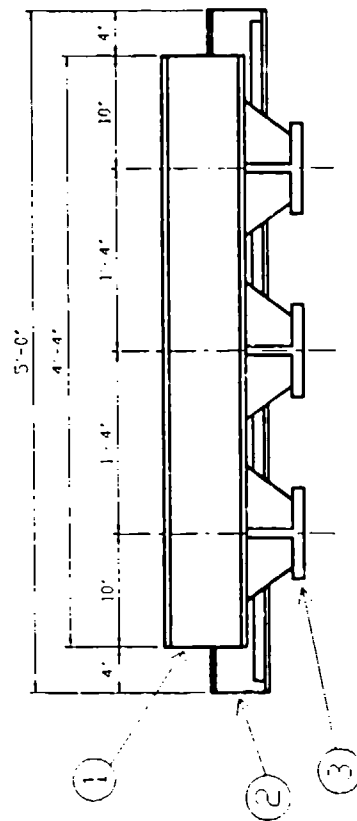


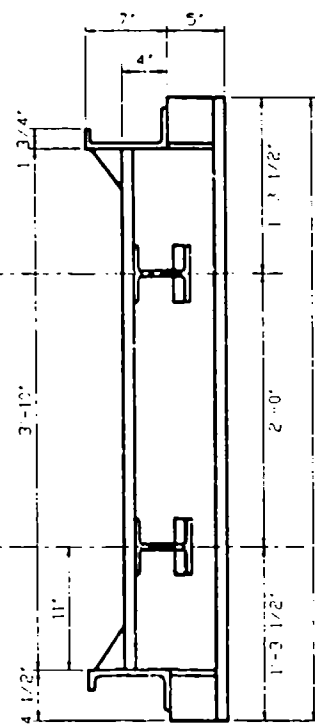
Figure B1 Sheet 3: Two DOF Fixture - Exploded View



See Detail Figure B2 Sheet 4

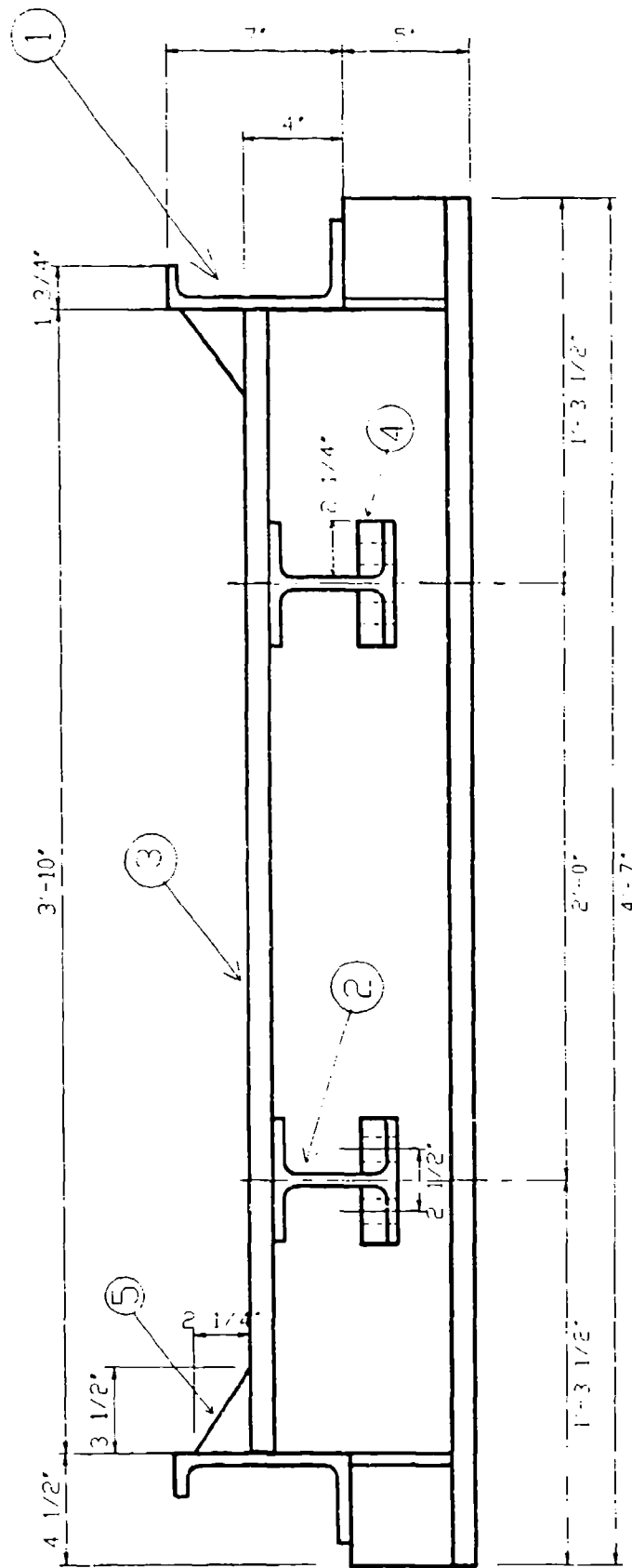


See Detail Figure B2 Sheet 3



See Detail Figure B2 sheet 2

Figure B2 Sheet 1: Intermediate Structure (M2)



Note: Piece 2 runs through piece 3 then fully welded in place

Figure B2 Sheet 2: Intermediate Structure Front View

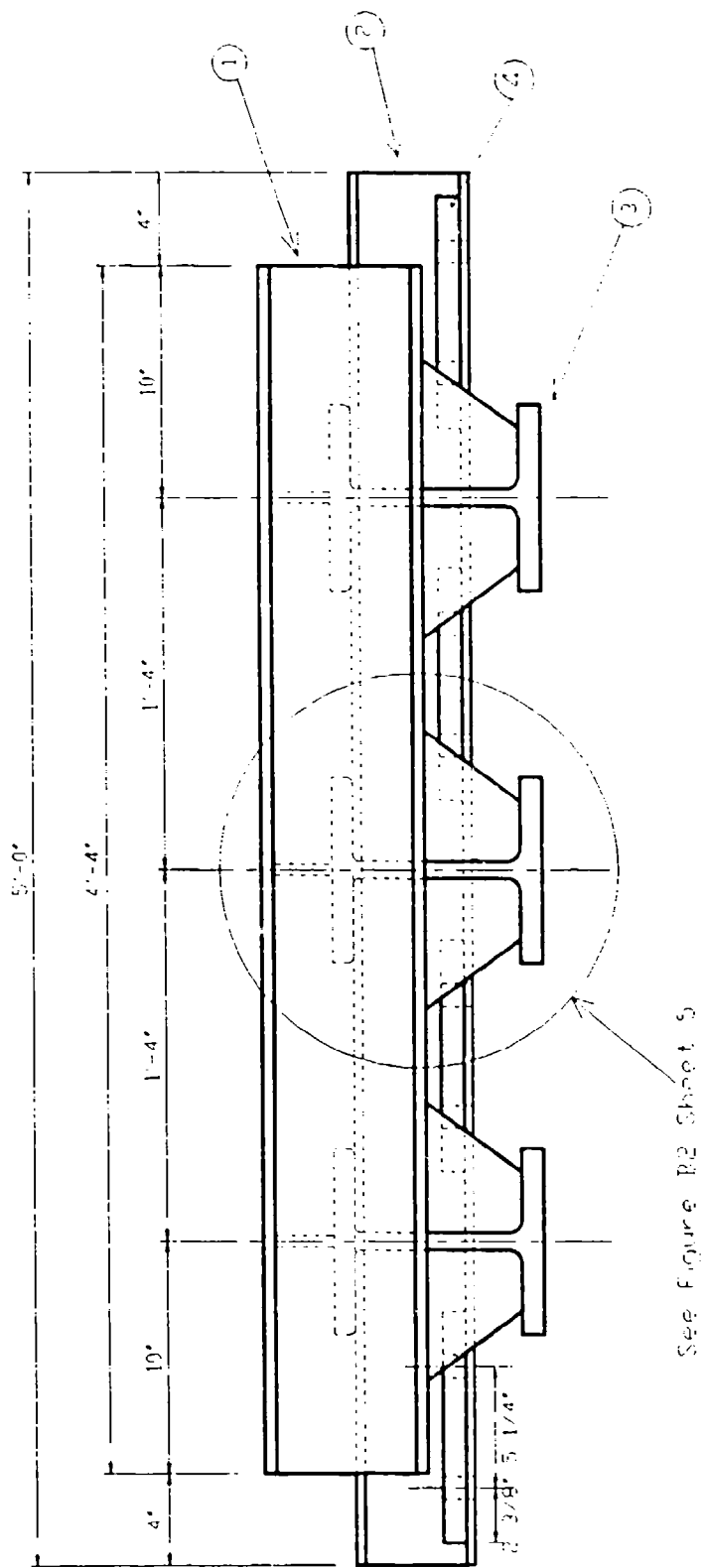


Figure B2 Sheet 3: Intermediate Structure Side View

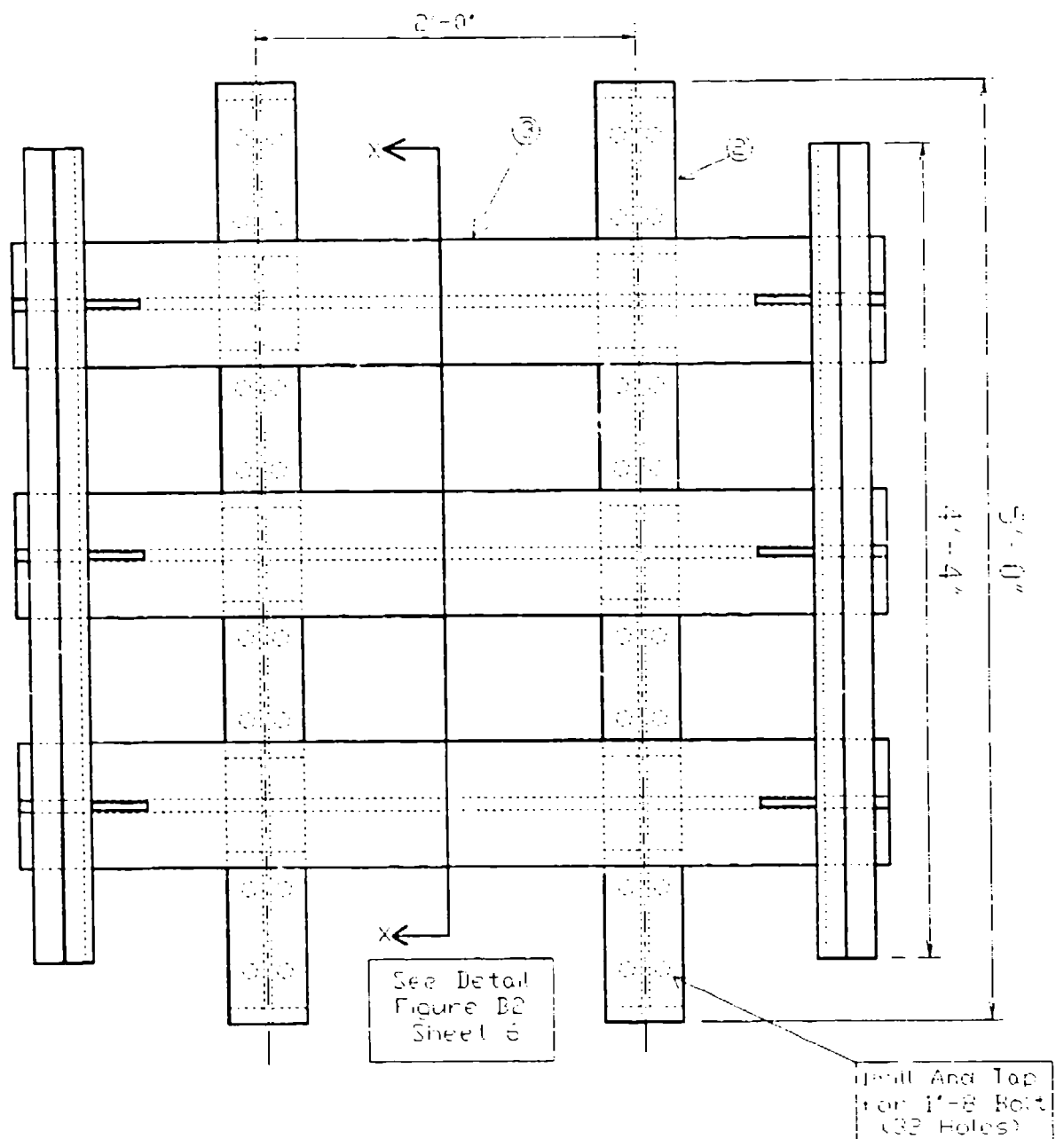


Figure B2 Sheet 4: Intermediate Structure Top View

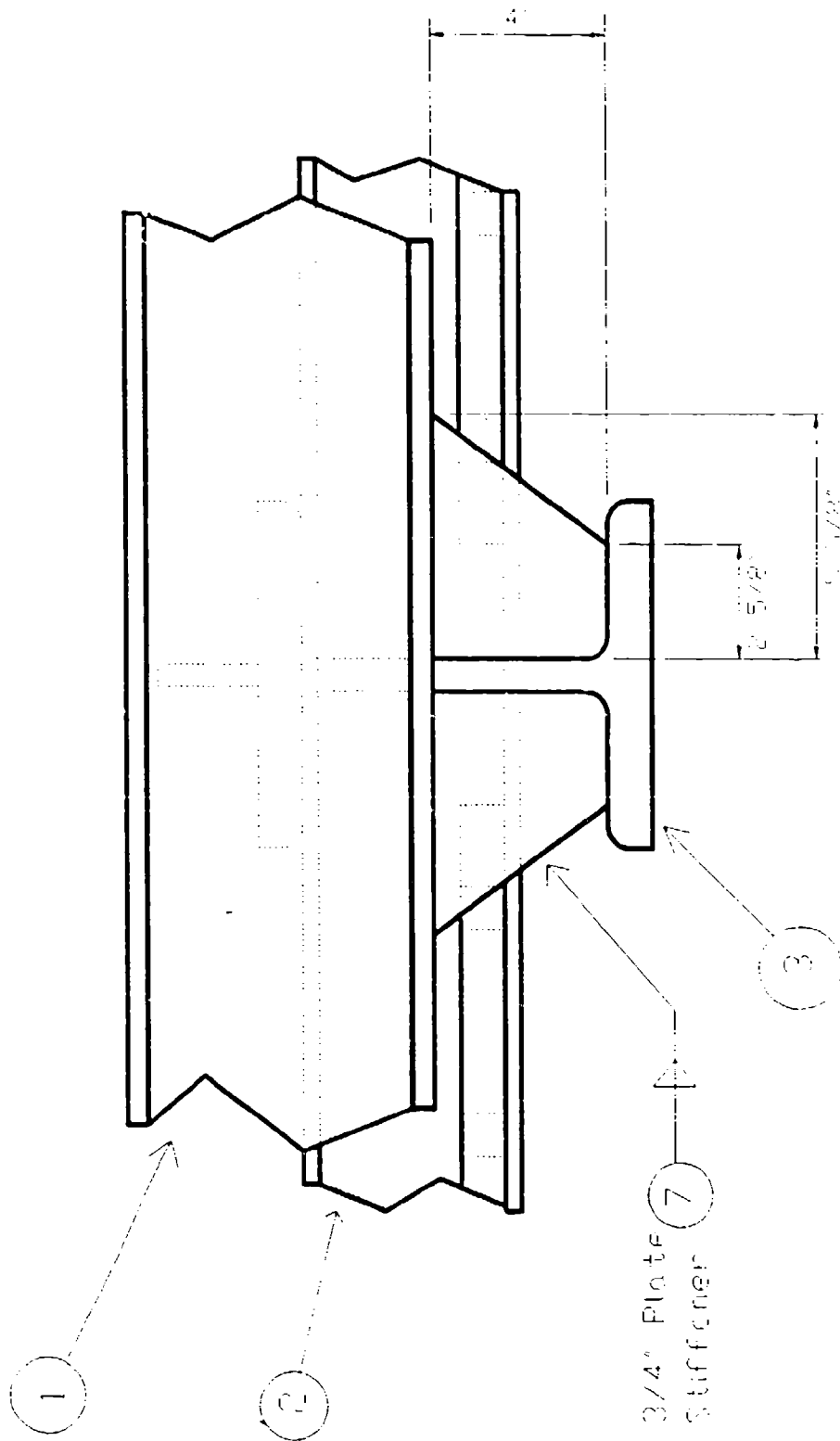


Figure B2 Sheet 5: Intermediate Structure Detail View

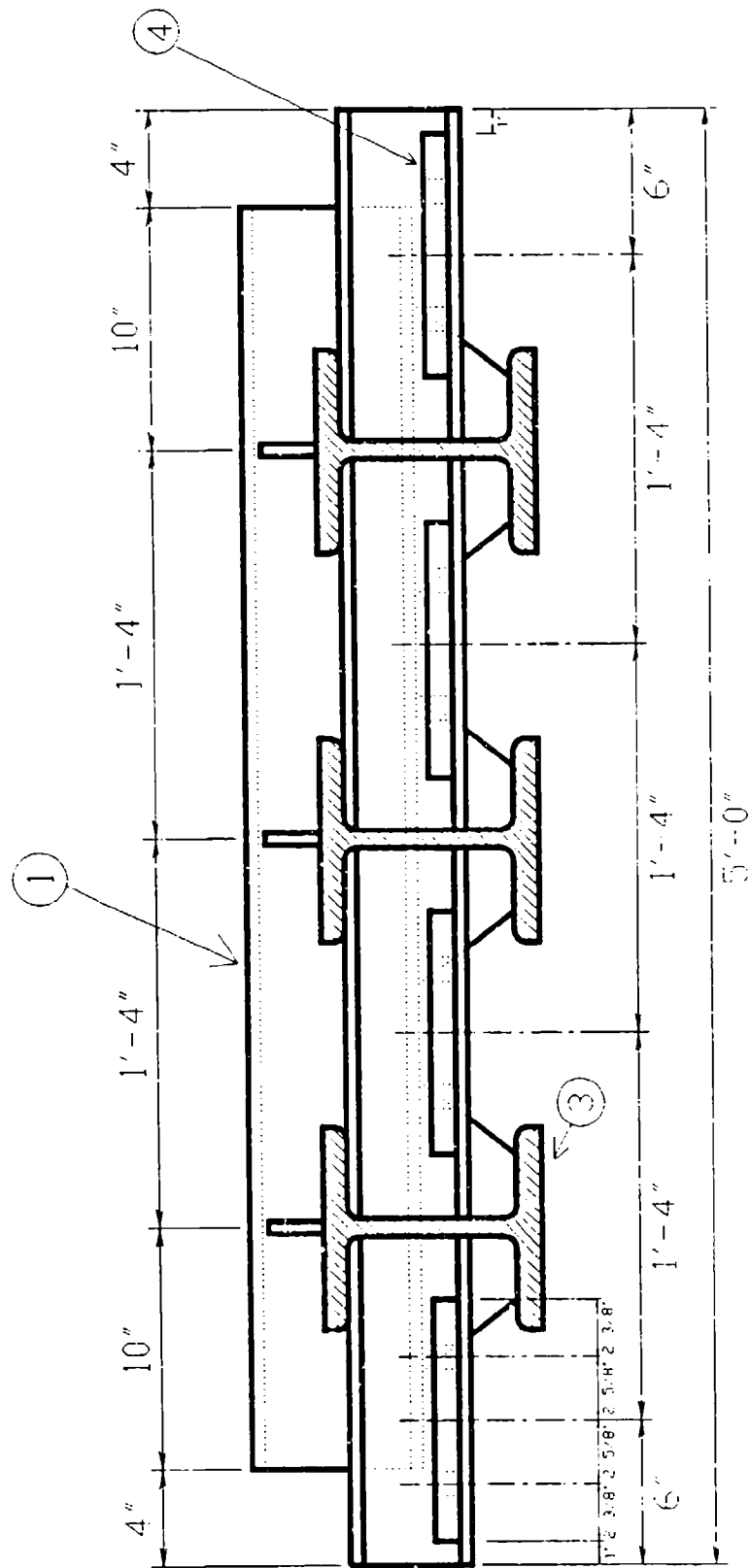


Figure B2 Sheet 6: Intermediate Structure Sectioned View

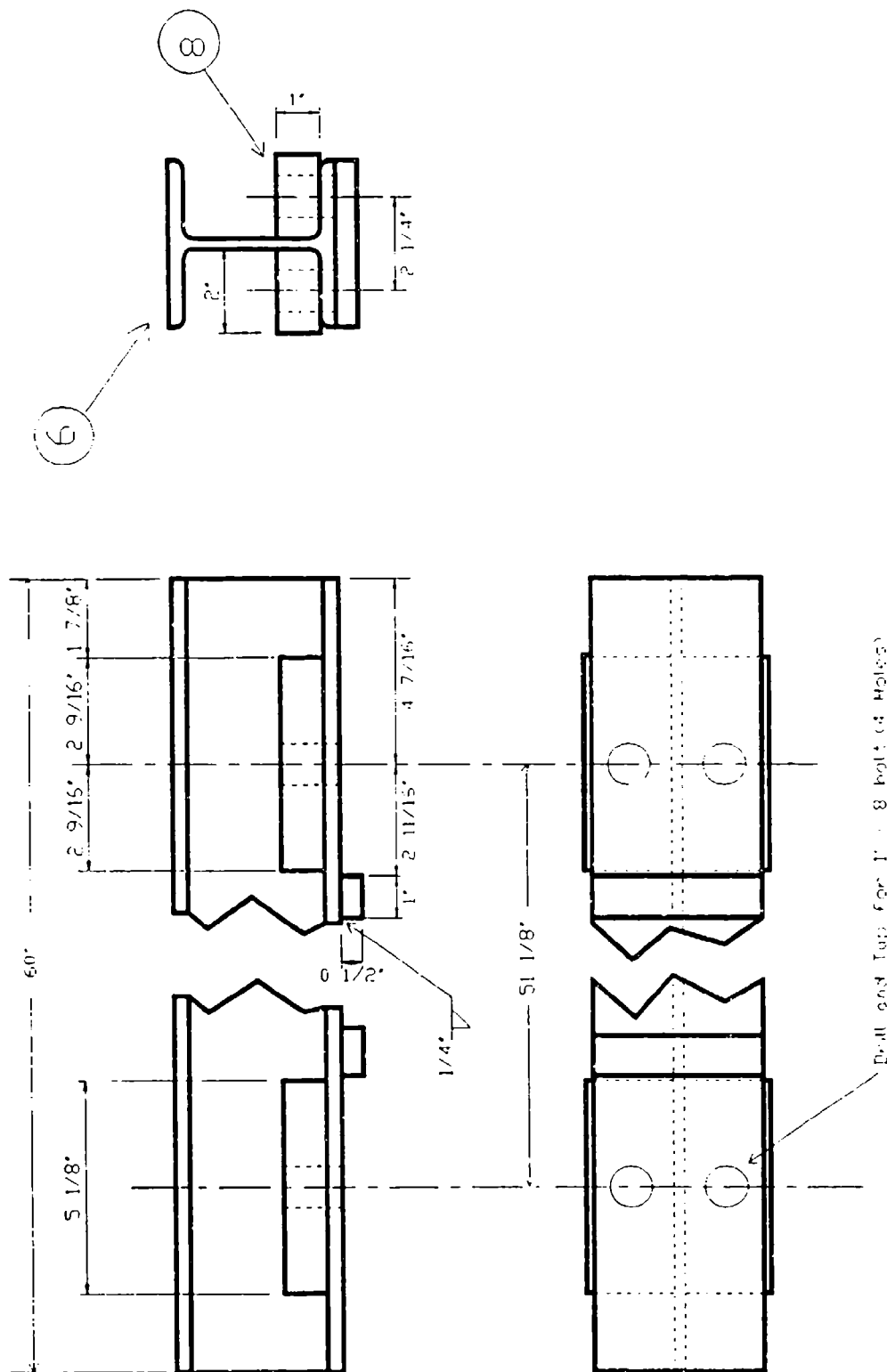


Figure B3 Sheet 1: Spring Beams

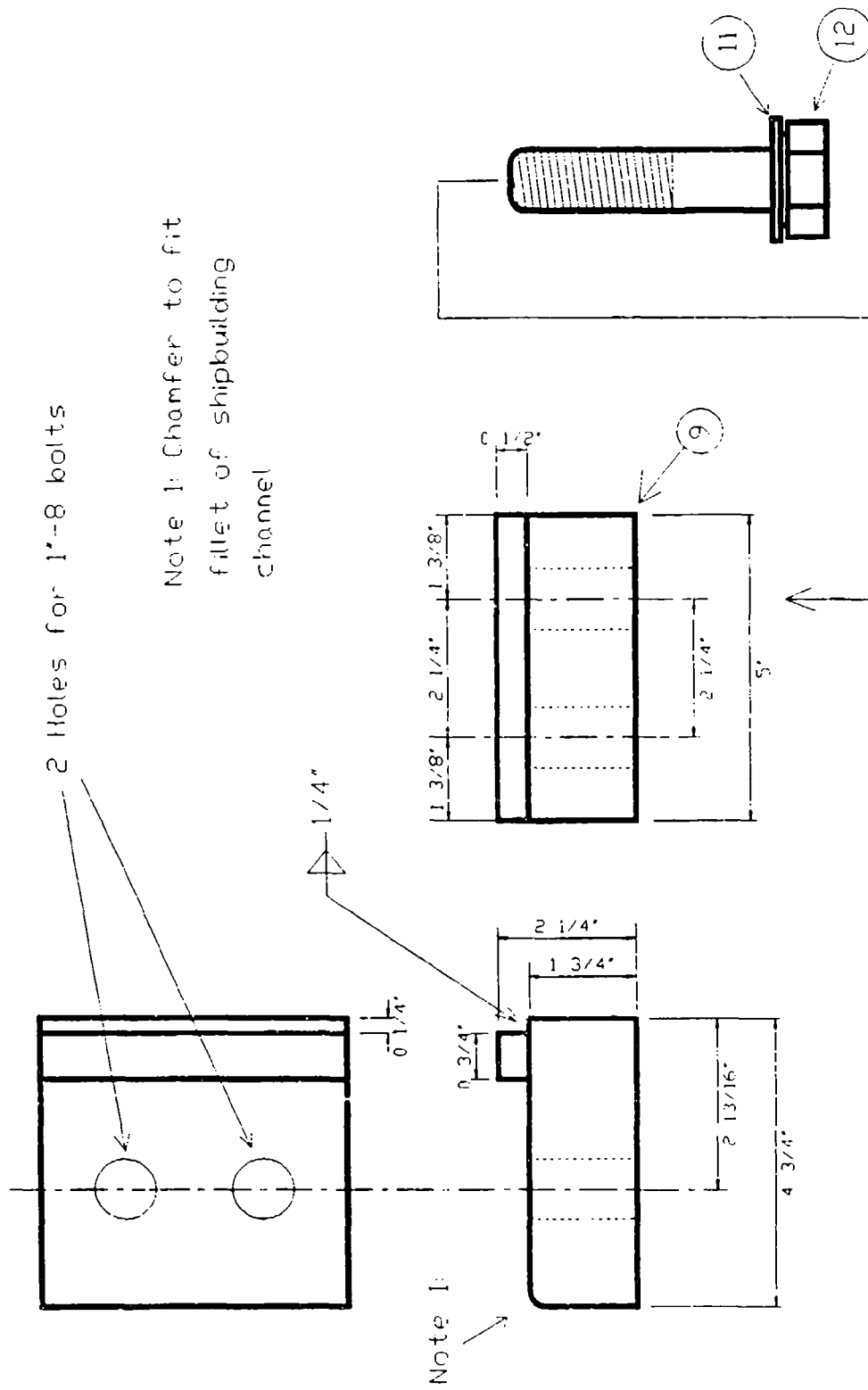


Figure B4 Sheet 1: Spring Beam End Clamp

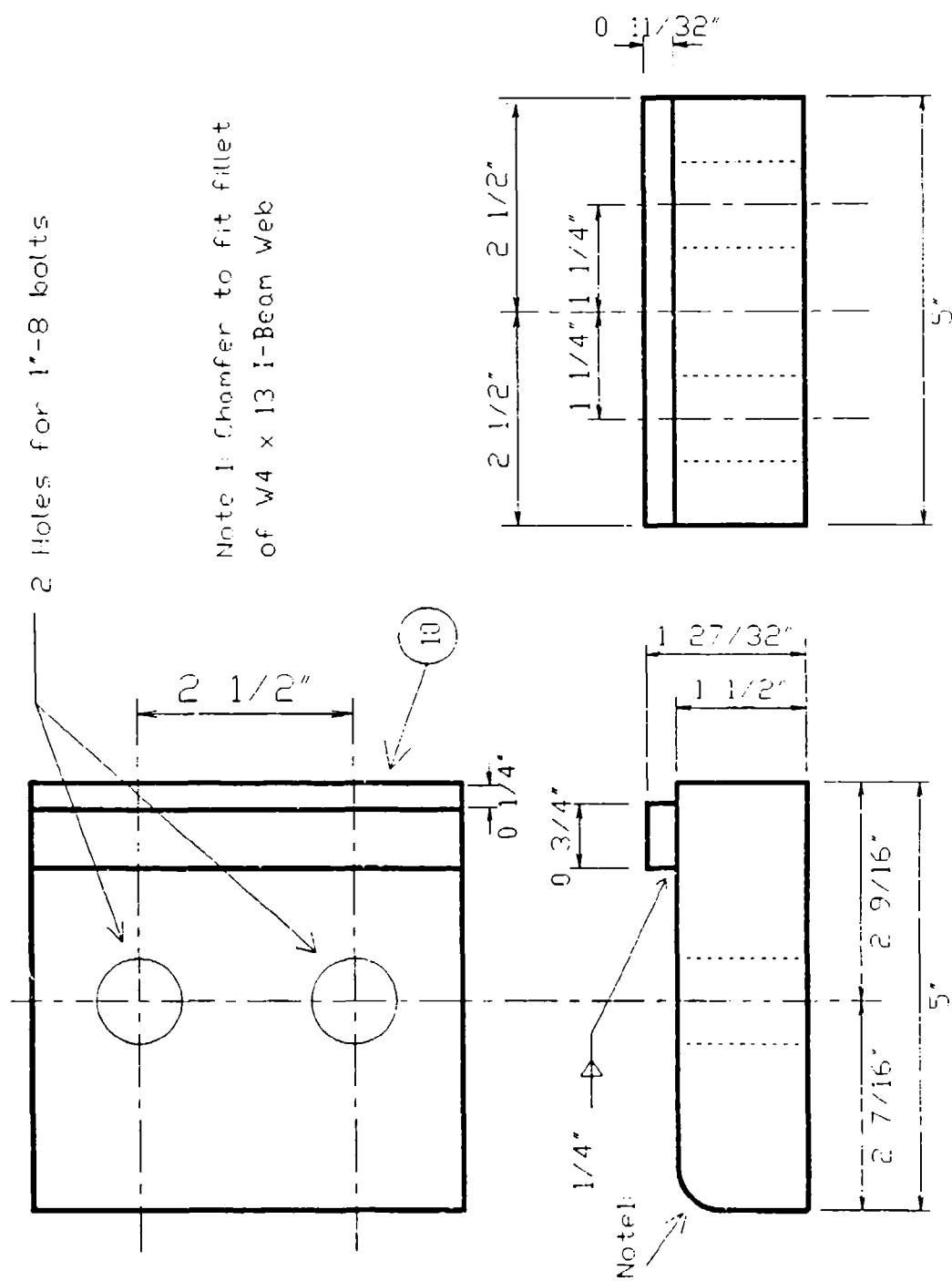


Figure B5 Sheet 1: Spring Beam Mid Clamps

Note: Mounting of equipment to I-Beams will be equipment specific

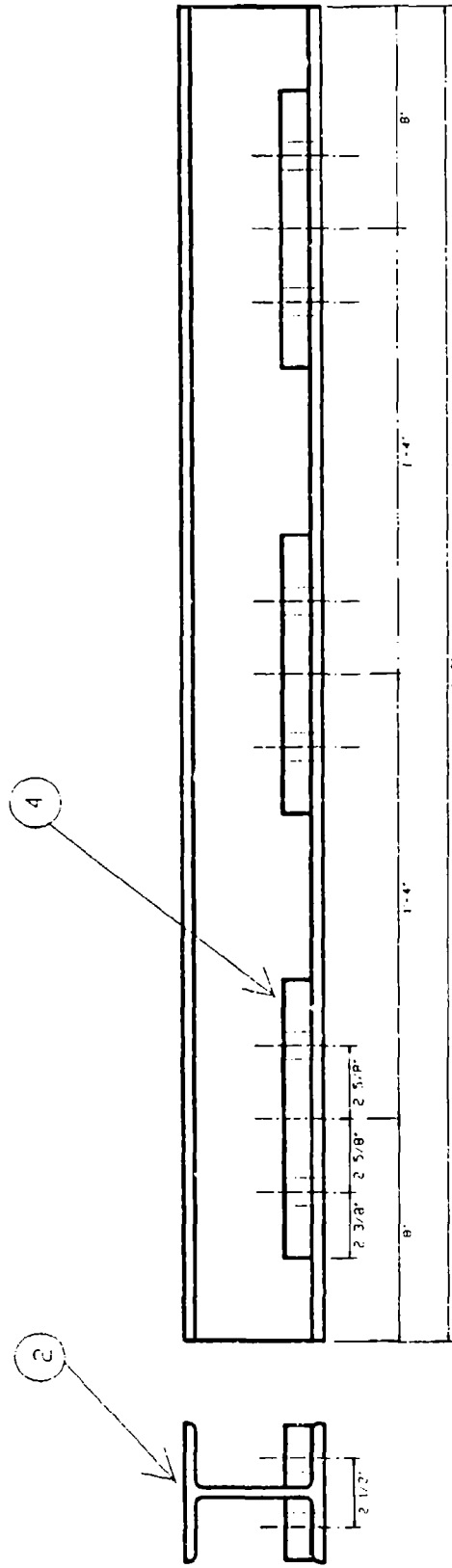


Figure B6 Sheet 1: Equipment Mount

APPENDIX C. MATHCAD® WORKSHEETS

APPENDIX C1

MathCad Worksheet To Find Two DOF Fixture Uncoupled Frequencies:

Input the Desired Coupled Natural Frequencies: (Hz)

$$f_{n1} = 30.00 \quad f_{n2} = 80.00$$

Input The Mass Ratio:

$$\text{Mass Ratio: } \alpha = 1$$

Initial Guess at uncoupled frequencies: $f_1 = 40 \quad f_2 = 40$

Given the initial guess for f_1 and f_2 , the following equations are iteratively solved for the required values of f_1 and f_2 .

Given

$$f_{n1} = \frac{1}{\sqrt{2}} \cdot \sqrt{(f_1)^2 + \alpha (f_1)^2 + (f_2)^2} \cdot \sqrt{\left[(f_1)^2 + \alpha (f_1)^2 + (f_2)^2 \right]^2 - (2 \cdot f_1 \cdot f_2)^2}$$

$$f_{n2} = \frac{1}{\sqrt{2}} \cdot \sqrt{(f_1)^2 + \alpha (f_1)^2 + (f_2)^2} \cdot \sqrt{\left[(f_1)^2 + \alpha (f_1)^2 + (f_2)^2 \right]^2 - (2 \cdot f_1 \cdot f_2)^2}$$

$$\begin{pmatrix} f_1 \\ f_2 \end{pmatrix} = \text{Find}(f_1, f_2)$$

SOLUTION

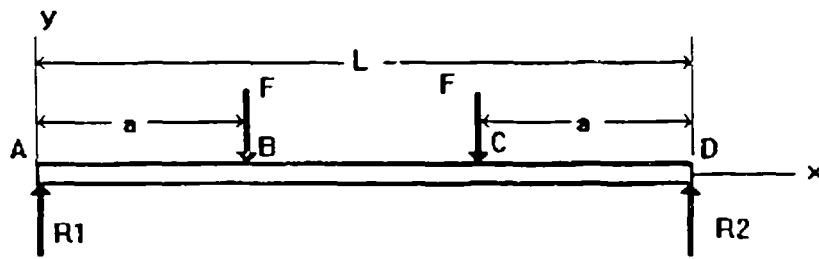
Required Uncoupled Tier Natural Frequencies: $f_1 = 49.963 \quad f_2 = 48.036 \quad \text{Hz}$

APPENDIX C2

Mathcad Worksheet to Determine System Natural Frequencies Of Two DOF Fixture:

A. SPRING STIFFNESS:

The spring stiffness is based on the following beam model:



Simple Supports - Symetric Loads

Supply the following Dimensions:

1) Dimensions:

	<u>Upper Tier</u>	<u>Lower Tier</u>
Length:	$L_1 = 50 \text{ in}$	$L_2 = 50 \text{ in}$
Load Application Point:	$a_1 = 13 \text{ in}$	$a_2 = 13 \text{ in}$
Area Moment of Inertia:	$I_1 = 11.3 \text{ in}^4$	$I_2 = 11.3 \text{ in}^4$
Young's Modulus:	$E = 30 \cdot 10^6 \text{ psi}$	$E = 30 \cdot 10^6 \text{ psi}$

2) Stiffness:

From the beam bending equation: $y_{AB} = \frac{F \cdot x}{6EI} \cdot (x^2 + 3 \cdot a^2 - 3 \cdot L)$

Let $x=a$ and $F=W/2$: $y_{AB} = \frac{W \cdot a^2}{12 \cdot E \cdot I} \cdot (4 \cdot a - 3 \cdot L)$

Solving for the stiffness: $K = \frac{W}{y_{AB}} = \frac{12 \cdot E \cdot I}{a^2 \cdot (3 \cdot L - 4 \cdot a)} \quad \frac{\text{lb}}{\text{in}}$

For:

	<u>Upper Tier</u>	<u>Lower Tier</u>
(Convert inches to feet)	$KB_1 = \frac{12 \cdot E \cdot I_1}{a_1^2 \cdot (3 \cdot L_1 - 4 \cdot a_1)} \cdot 12$	$KB_2 = \frac{12 \cdot E \cdot I_2}{a_2^2 \cdot (3 \cdot L_2 - 4 \cdot a_2)} \cdot 12$

(per beam)	$KB_1 = 2.947 \cdot 10^6 \frac{\text{lb}}{\text{ft}}$	$KB_2 = 2.947 \cdot 10^6 \frac{\text{lb}}{\text{ft}}$
------------	---	---

	<u>Upper Tier</u>	<u>Lower Tier</u>
Input Number of Beams Per Tier:	Beams ₁ = 2	Beams ₂ = 4
	$K_1 = KB_1 \cdot \text{Beams}_1$	$K_2 = KB_2 \cdot \text{Beams}_2$
Tier Stiffness:	$K_1 = 5.895 \cdot 10^6 \frac{\text{lb}}{\text{ft}}$	$K_2 = 1.179 \cdot 10^7 \frac{\text{lb}}{\text{ft}}$

B. SYSTEM MASSES: (Includes Effective Spring Mass):

Upper Tier Weight:	$W_1 = 2500 \text{ lbs}$	Lower Tier Weight:	$W_2 = 2500 \text{ lbs}$
	$M_1 = \frac{W_1}{32.2}$		$M_2 = \frac{W_2}{32.2}$
	$M_1 = 77.64 \text{ lb} \cdot \frac{\text{ft}}{\text{sec}^2}$		$M_2 = 77.64 \text{ lb} \cdot \frac{\text{ft}}{\text{sec}^2}$
Mass Ratio α :	$\alpha = \frac{M_1}{M_2}$		
	$\alpha = 1$		

C. TIER NATURAL FREQUENCIES:

<u>Upper Tier</u>	<u>Lower Tier</u>
$f_1 = \frac{1}{2 \cdot \pi} \sqrt{\frac{K_1}{M_1}}$	$f_2 = \frac{1}{2 \cdot \pi} \sqrt{\frac{K_2}{M_2}}$
$f_1 = 43.855 \text{ Hz}$	$f_2 = 62.02 \text{ Hz}$

D. SYSTEM NATURAL FREQUENCIES:

$$f_{n1} = \frac{1}{\sqrt{2}} \cdot \sqrt{(f_1)^2 + \alpha \cdot (f_1)^2 + (f_2)^2 - \sqrt{[(f_1)^2 + \alpha \cdot (f_1)^2 + (f_2)^2]^2 - (2 \cdot f_1 \cdot f_2)^2}}$$

$$f_{n1} = 33.565 \text{ Hz}$$

$$f_{n2} = \frac{1}{\sqrt{2}} \cdot \sqrt{(f_1)^2 + \alpha \cdot (f_1)^2 + (f_2)^2 + \sqrt{[(f_1)^2 + \alpha \cdot (f_1)^2 + (f_2)^2]^2 - (2 \cdot f_1 \cdot f_2)^2}}$$

$$f_{n2} = 81.033 \text{ Hz}$$

APPENDIX C3

Mathcad Worksheet to Find Two DOF Coupled System Natural Frequencies as a Function of the Uncoupled Tier Frequencies

Pick a Value For the Lower Tier Uncoupled Natural Frequency (F2): F2 = 48 Hz

Specify the Mass Ratio: $\alpha = 1.0$

Solve for the Coupled System Natural Frequencies as a Function of F1:

k = 1..100 F1_k = k

$$FN1_k = \frac{1}{\sqrt{2}} \sqrt{(F1_k)^2 + \alpha (F1_k)^2 + (F2)^2 - \sqrt{[(F1_k)^2 + \alpha (F1_k)^2 + (F2)^2]^2 - (2 F1_k F2)^2}}$$

$$FN2_k = \frac{1}{\sqrt{2}} \sqrt{(F1_k)^2 + \alpha (F1_k)^2 + (F2)^2 + \sqrt{[(F1_k)^2 + \alpha (F1_k)^2 + (F2)^2]^2 - (2 F1_k F2)^2}}$$

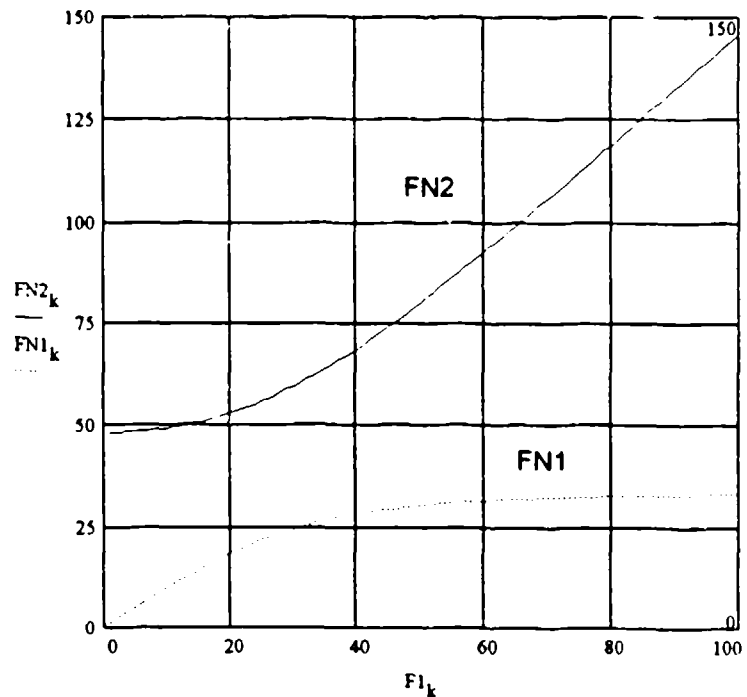
i = 40..60

PICK
UPPER TIER COUPLED

F1 _i	FN1 _i	FN2 _i
40	27.936	68.728
41	28.198	69.792
42	28.446	70.872
43	28.679	71.969
44	28.899	73.081
45	29.108	74.207
46	29.304	75.348
47	29.49	76.501
48	29.666	77.666
49	29.832	78.842
50	29.989	80.029
51	30.138	81.226
52	30.279	82.433
53	30.413	83.648
54	30.54	84.872
55	30.661	86.104
56	30.775	87.344
57	30.884	88.59
58	30.987	89.843
59	31.086	91.103
60	31.18	92.368

COUPLED SYSTEM NATURAL FREQUENCIES

$\alpha = 1$ F2 = 48



UPPER TIER NATURAL FREQUENCY

APPENDIX C4

MathCad Worksheet To Find Three DOF Fixture Uncoupled Frequencies:

Input the Desired Coupled Natural Frequencies: (Hz)

$$f_{n1} = 30.00 \quad f_{n2} = 80.00$$

Input The System Tier Weights: (lbs)

$$W_1 = 2500 \text{ lbs} \quad W_2 = 2500 \text{ lbs} \quad W_3 = 4500 \text{ lbs}$$

Mass Ratio:

$$\alpha = \frac{W_1}{W_2} \quad \beta = \frac{W_2}{W_3} \quad \gamma = \frac{W_1}{W_3}$$

$$\alpha = 1 \quad \beta = 0.556 \quad \gamma = 0.556$$

Supply the Program With An Initial Guess at f_1 and f_2 : $f_1 = 40 \text{ Hz}$ $f_2 = 40 \text{ Hz}$

Given the initial guess for f_1 and f_2 , the following equations are iteratively solved for the required values of f_1 and f_2 .

$$f_{n1} = \sqrt{\frac{1}{2} \left[(f_1)^2 \cdot (1 + \alpha) + (f_2)^2 \cdot (1 + \beta) - \sqrt{\left[(f_1)^2 \cdot (1 + \alpha) + (f_2)^2 \cdot (1 + \beta) \right]^2 - 4 \cdot (f_1)^2 \cdot (f_2)^2 \cdot (\beta + \gamma + 1)} \right]}$$

$$f_{n2} = \sqrt{\frac{1}{2} \left[(f_1)^2 \cdot (1 + \alpha) + (f_2)^2 \cdot (1 + \beta) + \sqrt{\left[(f_1)^2 \cdot (1 + \alpha) + (f_2)^2 \cdot (1 + \beta) \right]^2 - 4 \cdot (f_1)^2 \cdot (f_2)^2 \cdot (\beta + \gamma + 1)} \right]}$$

$$\begin{pmatrix} f_1 \\ f_2 \end{pmatrix} = \text{Find}(f_1, f_2)$$

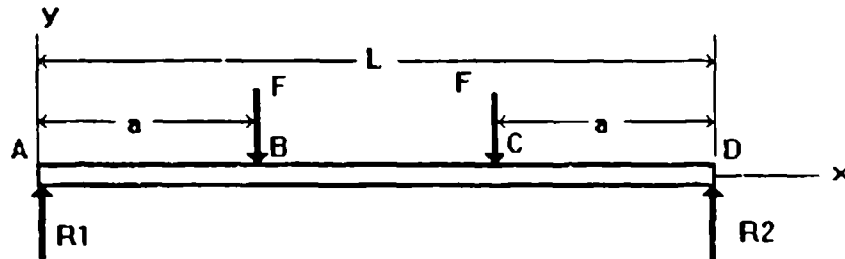
Required Uncoupled Tier Natural Frequencies: $f_1 = 54.077 \text{ Hz}$ $f_2 = 30.545 \text{ Hz}$

APPENDIX C5

Mathcad Worksheet to Determine System Natural Frequencies Of Three DOF Fixture:

A. SPRING STIFFNESS:

The spring stiffness is based on the following beam model:



Simple Supports - Symetric Loads

Supply the following Dimensions:

1) Dimensions:	<u>Upper Tier</u>	<u>Lower Tier</u>
Length:	$L_1 = 50 \text{ in}$	$L_2 = 50 \text{ in}$
Load Application Point:	$a_1 = 13 \text{ in}$	$a_2 = 13 \text{ in}$
Area Moment of Inertia:	$I_1 = 11.3 \text{ in}^4$	$I_2 = 11.3 \text{ in}^4$
Young's Modulus:	$E = 30 \cdot 10^6 \text{ psi}$	$E = 30 \cdot 10^6 \text{ psi}$

2) Stiffness:

From the beam bending equation: $y_{AB} = \frac{F \cdot x}{6 \cdot EI} \cdot (x^2 + 3 \cdot a^2 - 3 \cdot L)$

Let $x=a$ and $F=W/2$: $y_{AB} = \frac{W \cdot a^2}{12 \cdot EI} \cdot (4 \cdot a - 3 \cdot L)$

Solving for the stiffness: $K = \frac{W}{y_{AB}} = \frac{12 \cdot E \cdot I}{a^2 \cdot (3 \cdot L - 4 \cdot a)} \quad \frac{\text{lb}}{\text{in}}$

For:

	<u>Upper Tier</u>	<u>Lower Tier</u>
(Convert inches to feet)	$KB_1 = \frac{12 \cdot E \cdot I_1}{a_1^2 \cdot (3 \cdot L_1 - 4 \cdot a_1)} \cdot 12$	$KB_2 = \frac{12 \cdot E \cdot I_2}{a_2^2 \cdot (3 \cdot L_2 - 4 \cdot a_2)} \cdot 12$

(per beam)	$KB_1 = 2.947 \cdot 10^6 \frac{\text{lb}}{\text{ft}}$	$KB_2 = 2.947 \cdot 10^6 \frac{\text{lb}}{\text{ft}}$
------------	---	---

	<u>Upper Tier</u>	<u>Lower Tier</u>
Input Number of Beams Per Tier:	Beams ₁ = 2	Beams ₂ = 4
	$K_1 = KB_1 \cdot \text{Beams}_1$	$K_2 = KB_2 \cdot \text{Beams}_2$
Tier Stiffness:	$K_1 = 5.895 \cdot 10^6 \frac{\text{lb}}{\text{ft}}$	$K_2 = 1.179 \cdot 10^7 \frac{\text{lb}}{\text{ft}}$

B. SYSTEM MASSES: (Includes Effective Spring Mass)

<u>Upper Tier Weight: (W₁)</u>	<u>Intermediate Weight: (W₂)</u>	<u>Anvil Table Weight: (W₃)</u>
$W_1 = 2500$	$W_2 = 2500$	$W_3 = 4500$
$M_1 = \frac{W_1}{32.2}$	$M_2 = \frac{W_2}{32.2}$	$M_3 = \frac{W_3}{32.2}$
$M_1 = 77.64 \frac{\text{lb} \cdot \text{ft}}{\text{sec}^2}$	$M_2 = 77.64 \frac{\text{lb} \cdot \text{ft}}{\text{sec}^2}$	$M_3 = 139.752 \frac{\text{lb} \cdot \text{ft}}{\text{sec}^2}$

C. TIER NATURAL FREQUENCIES:

<u>Upper Tier</u>	<u>Lower Tier</u>
$f_1 = \frac{1}{2 \cdot \pi} \sqrt{\frac{K_1}{M_1}}$	$f_2 = \frac{1}{2 \cdot \pi} \sqrt{\frac{K_2}{M_2}}$
$f_1 = 43.855 \text{ Hz}$	$f_2 = 62.02 \text{ Hz}$

D. System Natural Frequencies:

Defining B as:

$$B = \frac{K_1}{M_1} + \frac{K_2}{M_3} + \frac{K_1 + K_2}{M_2}$$

$$f_{n1} = \frac{1}{2 \cdot \pi} \sqrt{\frac{1}{2} \left[B - \sqrt{B^2 - \frac{4 \cdot K_1 \cdot K_2}{M_1 \cdot M_2 \cdot M_3} (M_1 + M_2 + M_3)} \right]}$$

First Frequency: $f_{n1} = 44.64 \text{ Hz}$

$$f_{n2} = \frac{1}{2 \cdot \pi} \sqrt{\frac{1}{2} \left[B + \sqrt{B^2 - \frac{4 \cdot K_1 \cdot K_2}{M_1 \cdot M_2 \cdot M_3} (M_1 + M_2 + M_3)} \right]}$$

Second Frequency: $f_{n2} = 88.528 \text{ Hz}$

APPENDIX C6

Mathcad Worksheet to Find Three DOF Coupled System Natural Frequencies as a Function of the Uncoupled Tier Frequencies

Pick a Value For the Lower Tier Uncoupled Natural Frequency (f_2): $f_2 = 48 \text{ Hz}$

Input The System Tier Weights and calculate mass ratios:

$$W_1 = 2500 \text{ lb} \quad W_2 = 2500 \text{ lb} \quad W_3 = 4500 \text{ lb}$$

$$\alpha = \frac{W_1}{W_2} = 1 \quad \beta = \frac{W_2}{W_3} = 0.556 \quad \gamma = \frac{W_1}{W_3} = 0.556$$

Solve for the Coupled System Natural Frequencies as a Function of $F1$:

Define Mass/Stiffness Constant: $B_k = (f_{1k})^2(1 + \alpha) + (f_2)^2(1 + \beta)$ $k = 1 \dots 100 \quad f_{1k} = k$

$$f_{n1k} = \sqrt{\frac{1}{2} \left[B_k - \sqrt{(B_k)^2 - 4(f_{1k})^2(f_2)^2(\beta + \gamma + 1)} \right]}$$

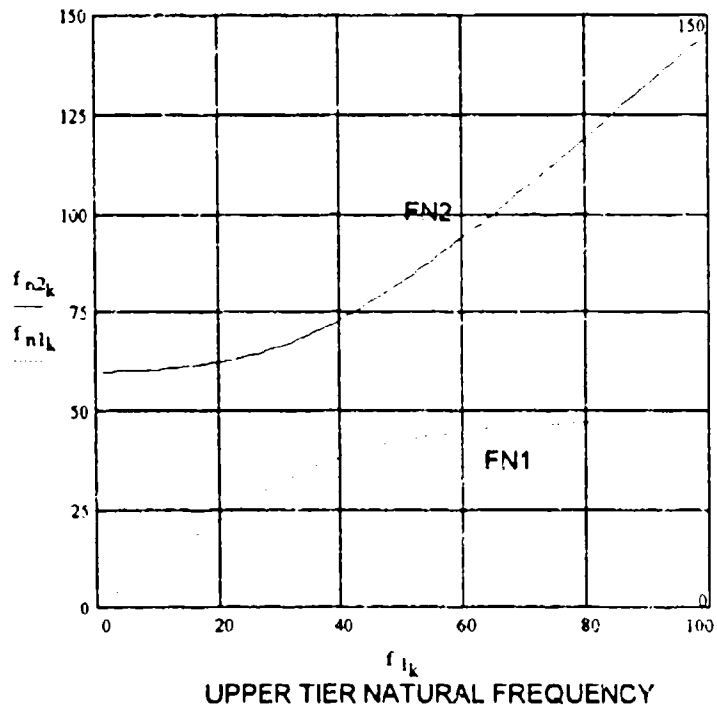
$$f_{n2k} = \sqrt{\frac{1}{2} \left[B_k + \sqrt{(B_k)^2 - 4(f_{1k})^2(f_2)^2(\beta + \gamma + 1)} \right]}$$

PICK $i = 30 \dots 50$
UPPER TIER COUPLED

f_{1i}	f_{n1i}	f_{n2i}
30	31.593	66.226
31	32.383	66.763
32	33.147	67.33
33	33.882	67.926
34	34.59	68.553
35	35.269	69.211
36	35.92	69.898
37	36.542	70.616
38	37.136	71.365
39	37.703	72.142
40	38.242	72.949
41	38.754	73.784
42	39.24	74.647
43	39.702	75.537
44	40.138	76.452
45	40.552	77.392
46	40.943	78.356
47	41.313	79.342
48	41.662	80.351
49	41.993	81.379
50	42.305	82.427

COUPLED SYSTEM NATURAL FREQUENCIES

$$\alpha = 1 \quad f_2 = 48$$



APPENDIX D. THREE DOF TUNED FIXTURE PROGRAM

```
% Program THREEDOF.M
% Lt. David M. Cox
% Naval Postgraduate School

% This program plots the relative displacement, relative
% velocity and absolute acceleration of a three DOF mass,
% spring, damper system subjected to base excitation (half-sine
% pulse). Additionally, the shock spectra of an undamped SDOF
% system is calculated using the upper tier acceleration as a
% base excitation to the SDOF system.

%User Defined Variables
HH=5 % Drop height in feet
Wt1=3000 % Weight of Upper Tier in Lbs
Wt2=2500 % Weight of Lower Tier in Lbs
Wt3=4500 % Weight of Anvil Table in Lbs
kbeam=2.947e6 % Stiffness of one Beam Lbs/ft
k1=3*kbeam % Upper Tier Stiffness in Lbs/ft
k2=4*kbeam % Lower Tier Stiffness in Lbs/ft
zeta=.04 % Critical Damping coefficient
g=32.2 % acceleration of gravity

%Formulate the Base Acceleration
dt=0.0001; % time step for sampling
N=5000; % Number of steps
Tblvel=sqrt(2*g*HH); % Velocity of Table after impact
PkAccel=11.35*Tblvel % From Figure 19
T=N*dt; % Total time record
t=[0:dt:(N-1)*dt]; % Time vector from t=0 to t=T
w=1000*pi;
for i=1:N % loop to produce half-sine wave pulse
    if t(i)<=0.001
        Force(i) = (Wt3)*PkAccel*sin(w*t(i));
    else
        Force(i)= 0;
    end
end

%Calculate the mass and mass ratio
m1=Wt1/32.2 ;
m2=Wt2/32.2 ;
m3=Wt3/32.2 ;
m=[m1,0,0;0,m2,0;0,0,m3];
```

```

    alpha=m1/m2;
    beta=m2/m3;
    gamma=m1/m3;
%Calculate Circular Tier Natural Frequencies
    w1=sqrt(k1/m1);
    w2=sqrt(k2/m2);
%System Natural Frequencies
    b=k1/m1 + k2/m3 + (k1 + k2)/m2;
    wn1=0
    wn2=sqrt(0.5*(b-sqrt(b^2-(4*k1*k2)*(m1+m2+m3)/(m1*m2*m3))))
    wn3=sqrt(0.5*(b+sqrt(b^2-(4*k1*k2)*(m1+m2+m3)/(m1*m2*m3))))

%Uncouple The Equations
    u11=1.0;
    u21=1.0;
    u31=1.0;
    u12=1.0;
    u22=(k1-m1*wn2^2)/k1;
    u32=(-k1 + (k1 + k2 - m2 * wn2^2)*u22)/k2;
    u13=1.0;
    u23=(k1-m1*wn3^2)/k1;
    u33=(-k1 + (k1 + k2 - m2 * wn3^2)*u23)/k2;
    U=[u11,u12,u13;u21,u22,u23;u31,u32,u33]
%Decouple the System
    M=U'*m*U
%Stiffness Matrix
    k=[k1,-k1,0;-k1,k1+k2,-k2;0,-k2,k2];
    K=U'*k*U
%Force Coefficients
    FC=U'*[0;0;1]
%Solve the Uncoupled Equations of Motion
% {nddot1}+[2*zeta*wn1^2]*{ndot1}+[wn1^2]*{n1}=FC11*Force/M1
% {nddot2}+[2*zeta*wn2^2]*{ndot2}+[wn2^2]*{n2}=FC21*Force/M2
% {nddot3}+[2*zeta*wn3^2]*{ndot3}+[wn3^2]*{n3}=FC31*Force/M3

%State Space Matrix for Equation #1
    A1=[0,1;-(wn1^2),-2*zeta*wn1];
    B1=[0;FC(1:1)/M(1:1)];
    C1=[1,0;0,1;-(wn1^2),-2*zeta*wn1];
    D1=[0;0;0;FC(1:1)/M(1:1)];
    [y1]=lsim(A1,B1,C1,D1,Force,t);

%State Space Matrix for Equation #2
    A2=[0,1;-(wn2^2),-2*zeta*wn2];
    B2=[0;FC(2:2)/M(5:5)];
    C2=[1,0;0,1;-(wn2^2),-2*zeta*wn2];
    D2=[0;0;0;FC(2:2)/M(5:5)];
    [y2]=lsim(A2,B2,C2,D2,Force,t);

```

```

%State Space Matrix for Equation #3
A3=[0,1,-(wn3^2),-2*zeta*wn3];
B3=[0;FC(3:3)/M(9:9)];
C3=[1,0;0,1,-(wn3^2),-2*zeta*wn3];
D3=[0;0;FC(3:3)/M(9:9)];
[y3]=lsim(A3,B3,C3,D3,Force,t);

%Couple the Equations
X1DIS = u11*y1(:,1) + u12*y2(:,1) + u13*y3(:,1);
X1VEL = u11*y1(:,2) + u12*y2(:,2) + u13*y3(:,2);
X1ACC = (u11*y1(:,3) + u12*y2(:,3) + u13*y3(:,3))/g;

X2DIS = u21*y1(:,1) + u22*y2(:,1) + u23*y3(:,1);
X2VEL = u21*y1(:,2) + u22*y2(:,2) + u23*y3(:,2);
X2ACC = (u21*y1(:,3) + u22*y2(:,3) + u23*y3(:,3))/g;

X3DIS = u31*y1(:,1) + u32*y2(:,1) + u33*y3(:,1);
X3VEL = u31*y1(:,2) + u32*y2(:,2) + u33*y3(:,2);
X3ACC = (u31*y1(:,3) + u32*y2(:,3) + u33*y3(:,3))/g;

plot(t,X1ACC),grid
xlabel('TIME (sec)'),ylabel('ACCELERATION (g''s)')
title('MODELED THREE-DOF FIXTURE UPPER TIER ACCELERATION')
gtext('5 Foot Hammer Drop (200 g''s)')
gtext('Zeta = 0.04')
gtext('Upper Tier Weight = 3500 lb')
gtext('Lower Tier Weight = 2700 lb')
gtext('Anvil Weight = 4500 lb')
meta fig33.met
pause

%Upper tier velocity
plot(t,X1VEL),grid
xlabel('Time (sec)'),ylabel('Velocity - ft/sec')
title('PREDICTED 3-DOF MODEL UPPER TIER VELOCITY')
gtext('W1=43.5 lbs; W2=57 lbs; W3=109 lbs')
gtext('Drop Height=6 in; Zeta=0.02')
meta threedof.met
pause

%Relative Displacements
plot(t,(X3DIS-X1DIS)*12,t,(X3DIS-X2DIS)*12),grid
title('PREDICTED RELATIVE DISPLACEMENT WITH RESPECT TO ANVIL TABLE')
xlabel('Time (sec)'),ylabel('Displacement (in)')
gtext('W1=43.5 lbs; W2=57 lbs; W3=109 lbs')
gtext('Drop Height=6 in; Zeta=0.02')
gtext('Upper Tier'),gtext('Lower Tier')
meta threedof.met
pause

```

```

%Calculate the shock spectra
NF=120;
SF=1;
DF=1;
Freq=[SF:DF:NF];
wn=2*pi*Freq;
F=[0;-1];
H=0;
for i=1:NF
    E=[0,1;-(wn(i)^2),0];
    GG=[-(wn(i)^2),0];
    [spec]=lsim(E,F,GG,H,X1ACC,t);
    maxspec(i)=max(abs(spec));
end

plot(Freq,maxspec),grid,xlabel('FREQUENCY (Hz)')
ylabel('ACCELERATION (g''s)')
title('MODELED THREE DOF FIXTURE ACCELERATION SHOCK SPECTRA')

```

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